INVESTIGATING BHUTANESE MATHEMATICS TEACHERS’ BELIEFS AND PRACTICES IN THE CONTEXT OF CURRICULUM REFORM

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Abstract

The introduction of a new curriculum potentially challenges teachers’ beliefs and practices about teaching. This single embedded case study explores these challenges in the context of a new mathematics curriculum in Bhutan where major educational reform has been undertaken. Limited previous research on mathematics education has been conducted in Bhutan, and limited research is available that explores the issue of the adoption of Western curriculum and its assumptions in a developing country. Thus the aim of this study was to investigate and understand the beliefs and practices of Bhutanese primary teachers implementing a new mathematics curriculum. This new curriculum was based upon principles and standards taken from a Western education context in a non-Western culturally distinct context. Of particular focus were the epistemological theories that Western curricula currently emphasise as a foundation of learning, namely those based on socio-constructivism. In such theories, meaningful learning occurs when students engage actively with concepts and problems in a dialogic process where ideas are shared, discussed and assimilated.

The phenomenon of implementing a new mathematics curriculum was explored in two phases drawing on sequential explanatory mixed-method approaches. First, in the macro level phase, a multi-mode survey was conducted with 80 respondents from 40 randomly selected primary schools across the country. The survey combined demographic questions, traditional theory driven belief items (based on Likert scales) and an open ended response involving the design of a sample learning activity. The belief items were designed based on the instrument of Perry, Howard, and Tracey (1999). The resulting data served to construct a background picture regarding the beliefs and practices of primary school mathematics teachers in Bhutan. Second, in the micro level phase of the study, qualitative data were collected from the teachers of four sections of Class 5 students (age 10-11) at two government primary schools. These data comprised teachers’ lesson plans, classroom observations and teacher reflections gathered during the teaching of a unit on fractions.
In regards to teachers’ beliefs, the micro level survey indicated a moderate to strong endorsement of reform oriented views of mathematics and mathematics education. However, the survey’s open ended responses indicated teachers tended to focus on what they do and not on what students do. This suggested an implied authoritarian conception of teaching mathematics and that teachers are in possession of knowledge that can be transferred through monologues of talk. Further micro level data from the teachers’ lesson plans and observations confirmed these findings from the survey and indicated a broad uniformity of approach in teaching mathematics, which provides evidence of their Platonist beliefs about mathematics.

Additionally, this study identifies some constraints in the implementation of the new curriculum, is a timely finding as the country is about to complete the first phase of the new curriculum. The study also confirms that underlying beliefs are strongly held by teachers despite enthusiastic acceptance of new initiatives in curriculum. Hence, the findings from this study are expected not only to help teachers to improve their knowledge and beliefs about meaningful implementation of mathematics lessons, but also to contribute to the further refinement of the mathematics curriculum. In this way, the findings from this study are expected to help contribute to improving the implementation of the new curriculum and also to promote the philosophy of Gross National Happiness by helping germinate a seed of new understanding in mathematics classrooms.
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List of Abbreviations

CAPSD: Curriculum and Professional Support Division
BCSE: Bhutan Higher Secondary Education Certificate
CISCE: Council for the Indian School Certificate Examinations
GNH: Gross National Happiness
MAD: Mathematics Activities Day
MoE: Ministry of Education
NAPE: New Approach to Primary Education
NCTM: National Council of Teachers of Mathematics
PTC: Primary Teacher Certificate
PP: Pre Primary
RME: Realistic Mathematics Education
ToT: Training of Trainers
STEM: Science, Technology Engineering and Mathematics
USDE: United States Department of Education
Statement of Original Authorship

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

QUT Verified Signature

Signature:

Date: 12 of May, 2016
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Kencho sum la Chag tsel lo!

(My homage and respect to the Triple Gems)
Chapter 1: Introduction

This thesis reports an investigation of the implementation of the new Bhutanese primary-level mathematics curriculum, which was introduced in 2008. A single embedded case study through an explanatory sequential mixed-methods approach was undertaken in two phases. The first phase, referred to as the macro level study provided a broad analysis of the general beliefs about mathematics and the implementation of the curriculum using a national survey of primary school teachers teaching mathematics. The second phase, referred to as the micro level study examined the teaching practices of five classroom teachers in two schools implementing one unit of the Class 5 (Year 5) mathematics curriculum. The overarching goal of the study was to investigate the beliefs and practices of Bhutanese primary school teachers implementing the philosophical and practical intentions advocated in the new curriculum.

This chapter presents a background and overview of the study: in Section 1.1, some general information regarding the educational system in Bhutan is given; Section 1.2 presents a profile of the researcher and her experience of mathematics and mathematics education in Bhutan; in Section 1.3, the Research aims and Research Questions are introduced; in Section 1.4, the methodological overview is presented; in Section 1.5, the significance of study is explained; and finally, the structure of thesis is presented in Section 1.6.

1.1 BHUTAN: THE CONTEXT

A brief background to the study is presented in this section, and is further detailed in Chapter 2. This study took place in Bhutan, the homeland of the researcher. Bhutan is known as the land of Gross National Happiness or GNH, to the outside world. Geographically, Bhutan is small in size but spiritually she is bigger than the largest country in the world. She belongs to the family of the Himalayan countries, sandwiched between two giant countries (China in the north and India in the south). Like only a few other countries, such as Sweden and Nepal, Bhutan has never been under foreign rule. Bhutan is a newly democratised country, having elected her first Prime Minister in 2008.
Politically, for centuries, Bhutan was ruled by twin leaders: the King as head of the administrative state, and Je-khembo the head of its spiritual body. This does not mean that Bhutanese people are God fearing: there is no God to worship. Bhutan has adopted the Tibetan Drukpa Kagyupa school of Buddhism since the 9th century. For Bhutanese people, leading a fair and selfless life is considered their true religion.

The fourth King started a process of governance reform, beginning in 2001 with the drafting of Bhutan’s first constitution, which was enacted in 2008. This was a most historic change in the organisation of the government. The nation’s aim, as defined in the constitution, includes ensuring literacy for the general public, and that students receive the highest quality of education (VanBalkom & Sherman, 2010).

Having benefitted from various scholarships to further my own learning, I felt a sincere obligation to repay this by conducting research to benefit the human development of the country. Bhutan is on the verge of development and therefore is in great need of human resources, mainly in relation to technology, for which knowledge of mathematics is crucial, for its contribution towards sustainable equitable socio economic development. Finally, success in this venture is expected to help individual teachers to thrive independently and help germinate a seed of GNH in mathematics classrooms as illustrated in Figure 1.1. Educating students mathematically is intended to contribute to the growth of GNH, in particular, one of the GNH pillars, sustainable and equitable socio economic development, as shown in Figure 1.1.

![Figure 1.1. GNH through mathematics education](image)

In Figure 1.1., two layers of information in terms of ideal development of mathematics education are presented with the outer layer (GNH, National Education...
Policy, social constructivist based mathematics curriculum and reformed oriented mathematics teachers) representing the aims, and the inner layer (Sustainable equitable socio economic development, Bhutanese mathematics education, four curriculum intentions and GNH in mathematics classrooms), representing the objectives. Each element in the outer layer is attached to its objective from the inner layer. For instance, although mathematical knowledge is applicable to all the four pillars of GNH, the broader philosophy of GNH is narrowed down to one of the four pillars (Sustainable equitable socio economic development). Similarly, the three remaining inner layers are drawn from the outer layers (mathematics education from national education policy, four curriculum intentions from social constructivist based mathematics curriculum and GNH in mathematics classroom guided by reformed oriented mathematics teachers). The arrow sign inside the circle is to indicate the flow of ideas in an anti-clockwise movement to maintain the relationship from one pair of elements to another, for instance, considering GNH as the main aim, developing education policy to produce a relevant mathematics curriculum and help germinate GNH in mathematics classrooms. In this sense, the overall aim of this specific model of GNH in Bhutanese mathematics classrooms is to help an individual to apply mathematical knowledge in terms of thinking globally and acting locally.

Of the four GNH pillars, sustainable equitable socio economic development is a key focus, mainly due to the shortage of a work force with high level mathematical knowledge in the country. Educating primary level students with a strong foundation mathematically is expected not only to help them make sense of their lives, but also to produce skilled professionals contributing to the socio economic development of the nation. However, educating school children in mathematics in Bhutan has been challenging, for reasons discussed in Section 2.3. Few researchers have attempted to conduct research on the quality of mathematics education in the country and so I felt a great need to conduct research in this area. In the following section, I present details of my experience with mathematics education in Bhutan.

1.2 THE PERSONAL CONTEXT FOR THIS STUDY
In discussing the role of a researcher in qualitative research, Malterud (2001) acknowledged that “a researcher's background and position will affect what they choose to investigate, the angle of investigation, the methods judged most adequate for this purpose, the findings considered most appropriate, and the framing and
communication of conclusions” (p. 483). To illuminate the researcher’s background and to position myself as the researcher within this study, the following brief personal background is provided.

Contributing to mathematics education in Bhutan has always been my burning desire. I have been working as the Dean of Student Affairs in Paro College of Education, one of the two colleges of education in Bhutan. I had been also heavily involved in teaching mathematics in the College since 1994. Prior to this post, I served in two different Secondary Schools in Bhutan as a mathematics teacher for about five years after completing my Bachelor’s Degree in teaching secondary school mathematics. This was followed by a year-long scholarship from the British Council to complete an Advanced Diploma in Mathematics Education from the University of Leeds, United Kingdom.

The course in Leeds was an eye opener for me in terms of my beliefs about the teaching and learning of mathematics. Based on new constructivist beliefs, I changed my teaching approach from a traditional teacher-centred method to activity-based learning of mathematics. My university students testify to my new approaches to teaching: one of my student teachers commented: “I would be in the medicine or engineering profession if I had mathematics teachers who taught like you in my school days”.

In 2002, I completed my Master’s by research in mathematics education from Edith Cowan University, Australia. This added another important feather to my researcher’s cap as I was the first Bhutanese student to achieve such a qualification. My exposure to mathematics education was further enhanced in 2005 with a Graduate Certificate in Educational Studies from the University of Newcastle, Australia. It was during those courses that I enriched my teaching practice through considering various potential teaching and learning theories in mathematics.

Besides sharing my new ideas and information with student teachers, I took initiatives to share ideas with my fellow mathematics teachers by organising a Mathematics Activities Day (MAD) in our College. I was also actively involved in writing guidebooks and textbooks for the new primary level mathematics, specifically from Classes PP to 6 (Pre-Primary–Year 6) and some of the mathematics modules for Bachelor of Education courses for the two teacher-training colleges in Bhutan. This opportunity, coupled with teaching experiences in the teacher-training
college, allowed me to explore ways to help students learn mathematics more productively. However, in this study, my main intention was to explore primary teachers’ beliefs, their practices and alignment with the philosophical intentions of the new curriculum. I believe, the quality of students’ learning can be inferred from the ways in which teachers conduct their lessons and their apparent beliefs about mathematics and mathematics education. Further, the idea of exploring teachers’ beliefs and practice was based on my anecdotal evidence of teachers’ difficulties in delivering the new and reformed mathematics curriculum. Based on this anecdotal evidence, I felt impelled to explore teachers’ implementation of the new curriculum.

The introduction of a nationally-framed mathematics curriculum was based on foreign models emanating from the US and other Western systems. For instance, the standards integrated into the new curriculum are mainly based on those of the National Council Teachers of Mathematics (NCTM), originating in the USA. This new curriculum was claimed to be dynamic in terms of helping students learn mathematics meaningfully and with deep understanding. However, the anecdotal evidence suggests that the apparent problem is the way in which it is being implemented in the mathematics classroom. With limited opportunity for professional learning activities related to the use of the new curriculum, it is questionable whether the pedagogical ideas intended in the new curriculum are actually being implemented. Consequently, there is a great need for this study to gather and analyse data from which it will be possible to create more awareness of the effectiveness of the new curriculum with the relevant authorities.

In the 50-year history of the modernisation of education in Bhutan, little research has been conducted into the effectiveness of the education system in general and into mathematics education in particular. So far, the few studies conducted on mathematics education have been minor and have been published in Rabsel, the journal of the Centre for Education and Research at Paro College of Education. Rabsel is the one of the few educational journals in Bhutan. Consequently, this study can cite very few articles specific to mathematics education in Bhutan. Hence, in addition to literature related to educational change in similar countries, when designing this study it has been necessary to draw upon the author’s experience-based authority, first as a learner, then as a classroom teacher, and currently as a
teacher educator in Bhutan. The aim of this study and its Research Questions are presented in the next section.

1.3 RESEARCH AIMS AND QUESTIONS

The overall aim of this study was to investigate and understand the beliefs and practices of Bhutanese primary teachers implementing the new mathematics curriculum. To address this aim, the following Research Questions were formulated:

Question 1: What are the beliefs of Bhutanese primary teachers about mathematics education?

Question 2: What are Bhutanese primary teachers’ planning and classroom practices in teaching mathematics?

Question 3: To what extent are mathematics teaching practices aligned with the curriculum intentions?

Question 4: What influences the implementation of the curriculum and its intentions?

The first Research Question intended to explore and explain the beliefs of teachers teaching mathematics in Bhutanese primary schools. The second Research Question sought to investigate and document the prevailing practices of primary school mathematics teachers. The third Research Question was about exploring the alignment between the philosophical intentions of the new curriculum and the actual practices conducted by teachers in the classroom. Building upon this examination of current practice, the fourth Research Question sought to explore the influences upon the teachers’ implementation of the curriculum and its intentions, in Bhutan.

An outcome of this study was to identify affordances and constraints for teachers in regard to implementing a new curriculum, thereby helping mathematics teachers to improve their practice and, ultimately, student learning outcomes. Further, findings from this study are expected to provide the evidence base needed for proposing effective professional learning initiatives in Bhutan. To address the four Research Questions, the methodology adopted is previewed in the following section.
1.4 METHODOLOGY OVERVIEW

The methodology adopted for this study was a single embedded case-study, with the case being teachers’ beliefs and practices subsequent to the implementation of a new mathematics curriculum in Bhutan. The design adopted for this study is Creswell, Clark, Gutmann, and Hanson (2003) explanatory sequential mixed method, further detail of which is given in Section 4.4. Teachers’ beliefs are considered important as the guiding principles for the conduct of teachers in the classroom, particularly in terms of teaching and learning of mathematics. The use of the mixed-method approach provided a broad overview of the phenomenon followed by more detailed investigation. The study was undertaken in two phases: macro-level and micro-level.

In the macro level phase, survey questionnaires were designed to explore in general the existing beliefs and practices of teachers teaching mathematics in Bhutanese Primary Schools, aligning with the philosophical intentions advocated in the new curriculum. The survey had 80 respondents from 40 randomly selected primary schools across the country. The survey combined demographic questions, traditional theory driven belief instruments (based on Likert scales) adapted from Perry et al. (1999), and an open-ended response involving the design of a sample learning activity by respondents. Analysis of the survey data was carried out to identify and understand beliefs espoused by teachers and their planning of learning activities in mathematics, so creating a background picture regarding their beliefs and practices.

To seek further clarification and explanations of the findings from the macro-level phase, the in-depth micro-level phase of the study was carried out to develop a deeper understanding of classroom practices and their alignment with the new curriculum intentions. This involved collection and analysis of lesson planning, classroom observation and teacher reflection data from a unit on fractions taught to several sections of Class 5 (age 10 -11) students at two different primary schools (Takin and Dragon). Five teachers (two from Takin and three from Dragon) were participants in this micro-level phase. The content topic of fractions was selected because of the researcher’s experience in teaching this topic. Moreover, it was based on her anecdotal evidence of Bhutanese teachers finding difficulties in teaching fractions, and her intention to be of some help to teachers during the process of data collection, at least those few teachers who were involved as participants in this study. To analyse the data, a priori coding was used based upon a framework aligning with
the stated intentions of the new curriculum. The teachers’ beliefs and practices were
examined. Thus, the detailed study of a few individuals aimed to paint a
comprehensive picture of the challenges facing mathematics education in Bhutan.

1.5 SIGNIFICANCE OF THE STUDY

The introduction of a new curriculum can potentially challenge teachers’ beliefs
about teaching mathematics. Limited research is available that explores the issue of
the integration of Western ideas in curriculum and its assumptions in a developing
country. The central phenomenon reported in this study is the implementation of a
reform-oriented curriculum, which was based on principles imported from Western
education contexts in a non-Western culturally distinct context. A particular focus is
a core epistemological theory that Western curricula emphasise as a foundation of
learning, namely social-constructivism as a theoretical lens. In this theory,
meaningful learning is considered to be taking place when students engage actively
with concepts and problems in a dialogic process where ideas are shared, discussed
and assimilated.

Despite attention to the quality of the mathematics curriculum for the
improvement of mathematics learning standards in the country, little consideration
has been given to its implementation. It is expected that this study will provide
contextually relevant evidence of the constraints and affordances confronted by
teachers implementing a new mathematics curriculum. It is possible that the
documentation of these teachers’ experiences and consequent findings might be used
to inform pre-service teacher education programs in training colleges, and also to
inform professional learning activities for in-service mathematics teachers in the
schools. Thus, this study may indirectly enhance the opportunities for learners to
experience learning mathematics in a more interesting and enjoyable manner, and
thereby contribute to the development of Bhutan’s GNH.

1.6 STRUCTURE OF THE THESIS

This thesis is organised into nine chapters. Following this introductory chapter,
Chapter 2 provides a more detailed description of the context of the study and the
need for this type of study to bring improvement in terms of teaching and learning of
mathematics. Chapter 3 reviews the literature on mathematics teachers’ beliefs,
constructivist-oriented views of teaching and learning of mathematics and issues
associated with curriculum reform and its implementation. Constructivism provides the philosophical basis of the new curriculum, which is based on NCTM (National Council of Teachers of Mathematics, 2000) standards. This review is summarised by the conceptual framework, which in turn is used to construct an analytical framework for the qualitative data.

A detailed description of the study’s methodology is provided in Chapter 4. The details of the results from the macro-level phase are presented in Chapter 5. The context and the results of the micro level phase are respectively presented in Chapter 6 and 7. The findings from both macro and micro level phases are presented in the form of discussion in Chapter 8, followed by the overall conclusion of the study in Chapter 9.
Chapter 2: Context

Bhutan is popularly known to the world as the land of Gross National Happiness (GNH). In this chapter, important aspects of the context of the study are described in five sections, all related to mathematics education in the country. This chapter extends the ideas introduced in Section 1.1. The foremost reason for including a separate chapter on context was to consider how the teaching and learning of mathematics has developed in Bhutan, as an historical and cultural context to help understand the current situation and its findings.

Section 2.1 provides a brief introduction to Bhutan, including geographical features, government structure, economy, and the guiding philosophy of Gross National Happiness. Section 2.2 provides a brief history of Bhutanese education in general and then focusses on the history of mathematics education and its recent reform. Central to this study is the alignment of teachers’ beliefs and classroom practices with the intent of the curriculum. Consequently, in Sections 2.3 and 2.4, the training and support for pre- and in-service teachers is discussed. Finally, Section 2.5 closes with a summary of the challenges faced by Bhutan in continuing her agenda of curriculum reform and highlights the role that this study might play in that reform.

2.1 NATIONAL CONTEXT

Bhutan is a small and beautiful landlocked country, located in the eastern Himalayas between China in the north and India in the south. It has an area of 38 394 square kilometres extending 300 kilometres east to west and 170 kilometres north to south. Bhutan is similar in size to Switzerland (39 777 km²) and slightly larger than the Netherlands. Compared to such similarly sized countries, the population of Bhutan is very small, with approximately 708,265 citizens (National Statistics Bureau, 2013). Around 72% of the country is covered with forest and only 3% is cultivated, with the remainder either barren, rocky or scrubland. Bhutan has one of the most rugged mountain terrains in the world, with elevations of more than 7000 metres above sea level (National Statistics Bureau, 2011). These conditions make physical and internet communications difficult.
The country is governed through three levels of administration, with the King as a constitutional monarch being the overall head of the nation. These three levels are:

- Central Government, with 10 different ministries each headed by a Minister;
- *Dzongkhag* (Districts), 20, each headed by a *Dzongdag* (District administrator);
- The *Gewog* (Blocks), 205, each headed by a *Gup* (Village headman).

There has been stable delegation of decision making and institution building since His Majesty Jigme Singye Wangchuk, the fourth King, who was enthroned in 1974. His Majesty’s philosophy of Gross National Happiness has guided Bhutan into the 21st century. The wish of the fourth King was to encourage the people to strive for Gross National Happiness rather than *Gross National Product*. This holy vision of His Majesty was intended to be put into practice through the four pillars of GNH which are summarised in Table 2.1. The main goal of adopting the four pillars was to shape the country to be economically prosperous, environmentally sustainable, well governed and culturally vibrant (Ministry of Education Bhutan, 2009).

**Table 2.1**

*Four Pillars of Gross National Happiness (GNH)*

<table>
<thead>
<tr>
<th>GNH Pillars</th>
<th>Aims and objectives</th>
</tr>
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<tbody>
<tr>
<td>Good governance</td>
<td>Dedicated to instituting a system of governance that upholds welfare and happiness of its citizens. Making every exertion to serve the people with morality, responsibility and transparency.</td>
</tr>
<tr>
<td>Preservation and promotion of culture</td>
<td>Instil in the heart of people the importance of preserving ones’ own beliefs and customs, which are considered a fundamental value for harmonious coexistence between humankind and nature.</td>
</tr>
<tr>
<td>Conservation of environment</td>
<td>Educating people on the importance of saving the existing beautiful and natural environment. Doing this will not only assist the country in the field of energy and tourism but also impact on the better quality of life in the future.</td>
</tr>
<tr>
<td>Sustainable and equitable socio-economic development</td>
<td>Motivating the people to be able to stand on their own and reduce dependences on external sources as much as possible to promote balanced socio economic development. In Bhutan’s exertion towards the attainment of GNH, the country does not discard economic expansion. Rather, there is a balance between economic development and spiritual traditions that is pursued.</td>
</tr>
</tbody>
</table>

Source: Ministry of Education, 2009
Modelling the pillar of good governance, in 1998 His Majesty delegated the King’s executive power to the Council of Ministers. This became the most historic change in the organisation of Bhutanese government. In addition to this, in 2001, the King drafted Bhutan’s first constitution, which was enacted in 2008. Thus the King modelled good governance by providing an effective and complete transition of national policy to a democratic constitutional monarchy. His Majesty abdicated the throne in favour of the Crown Prince, His Majesty Jigme Khesar Namgyal Wangchuck, in late 2008. Unlike the case in many other countries, Bhutanese people have not had to strive for democracy; rather it came as a gift from the King. In the same way, to date and despite its size, Bhutan is fortunate in having been and remaining free from external political influence. It is a matter of great pride for the Bhutanese to live in such a peaceful country.

Every year, thousands of tourists fly in to experience their share of ‘happiness’. The main attraction for tourists is Bhutan’s well preserved culture and environment. As a result, one of the main sources of income is from tourism. Hydro-electric power generation and agricultural products are other major sources of income. The change in the economy of Bhutan is a direct result of greater participation in the world; now her economy is expanding to include tourism, energy and many other industries. Bhutan still depends on other countries such as India for processed materials and labourers for construction activities. Nevertheless, one of the GNH pillars, sustainable and socio economic development has been set in train as one of the major policy initiatives and it is expected to play a significant role in the future economic growth of the country. This brief introduction to the country and its governance reflecting the GNH philosophy provides the basis for describing the significance of education in Bhutan, which is the focus of the remaining sections of this chapter.

2.2 A BRIEF HISTORY OF EDUCATION IN BHUTAN

This section firstly provides an overview of the history of education in Bhutan, before outlining particular aspects including a detailed description of the new mathematics curriculum.

Until 1950s, education in Bhutan was primarily monastic and literacy was limited to the monasteries (Verma & Dorzi, 2014). Bhutan has had a well-established Buddhist monastic education system dating back to the eighth century and many
renowned Bhutanese scholars travelled to Tibet to study Buddhist scriptures in subjects such as astrology, mathematics, medicines and choeki, the classical religious language (Mackey, 2013; Verma & Dorzi, 2014). In the 1950s, Bhutan opened its first secular schools with both the curriculum and the medium of instruction in Hindi borrowed from India (Mackey, 2013; Verma & Dorzi, 2014). Gradually, Western education influences were initiated by British contact through India but more recently by Christian missionaries. Father William Mackey, from Canada, served in the Ministry of Education from 1963 to 1995 in various positions (mathematics teacher, school principal and chief to school inspectors). He was popularly known as the ‘son of modern Bhutan’ and passed away as a true citizen of Bhutan. I was one of the lucky few who studied under the principalship of Fr. Mackey and was also a mathematics student of his in Classes 7 and 8 (Class = Year level).

The Buddhist monastic education system still plays an important role in the Kingdom’s social and religious life. The first school that functioned in a more Western way was established in Haa (western Bhutan) in 1914. Prior to this, there existed a mobile court school that moved around the country with the royal entourage of the first King (His Majesty Ugyen Wangchuck), who initiated the idea. The introduction of a modern general public education system in Bhutan came with the economic development initiated in 1961 under the dynamic leadership of His Majesty Jigme Dorji Wangchuk, the third King (Fricot, 2009). At the same time, English became the language of instruction in Bhutanese schools with the aim of helping Bhutan participate in the modern world (Dorji, 2005).

In 2010 the Ministry of Education initiated Educating for GNH as an integral part of school education to express and fulfil the fourth King’s ideal regarding Sem Gochoep Zoni or mindfulness education. The main intention behind this philosophy is to train students to look within themselves and become conscious of their thoughts and reactions, learning to be observant of their engagement in body, speech and mind (Thinley, 2013). Education is considered the framework for the overall development of conformity with the cultural and spiritual values of the country. It prepares the young with appropriate knowledge, skills, attitudes and values (thag-dhamtsig and lay ju-drey) that are required to achieve the goals of GNH (UNESCO, 2010). The concepts of tha-dhamtsig (sacred commitment to others) and lay-judrey (actions have consequences) are vital to Bhutanese values (Ministry of Education Bhutan, 2009).
The notion of wholesome education refers to the overall development of the child in terms of the cultural and spiritual values of the country. The Bhutanese education system aims to create opportunities for a person to grow up to be a true human being, with some sense of contributing to society in terms of generating self-sufficiency, productivity, contentment, peace and happiness. The entire underpinning of the school curriculum is based on such Bhutanese culture and values to achieve the objectives of GNH. The Bhutanese education system focusses on a deep and genuine care and respect for others, for nature, and for Bhutan’s profound and ancient culture. To achieve this goal, the government, via the Bhutanese Ministry of Education, aims to provide a minimum of 11 years of free quality education to all citizens (Ministry of Education, 2007). This includes not only free tuition, but also the provision of textbooks, stationery, meals and boarding facilities as required.

The present formal education system is structured in chronological grades as presented in Table 2.2, in which learners require certification to progress (National Statistics Bureau, 2011). The formal education system starts at the age of six years, when children are admitted into Pre-Primary (PP) classes. There are nine years of basic education: one year pre-primary, six years primary, two years junior high, followed by four years of secondary education (Gyamtso & Dukpa, 1999). Admittance from one level to the other is merit-based and determined by national and external examinations, as well as by the human resource plans and the space available at the relevant levels of education. This system in Bhutan particularly applies to class levels 6, 8, 10 and 12 as indicated in Table 2.2.

Table 2.2
Sequence of Bhutanese education

<table>
<thead>
<tr>
<th>Types of Education</th>
<th>Duration in terms of years</th>
<th>Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Primary School (PP)</td>
<td>1 year (Kindergarten)</td>
<td>Class VI summative</td>
</tr>
<tr>
<td>Primary School (PS)</td>
<td>6 years (Classes I – VI)</td>
<td>assessment</td>
</tr>
<tr>
<td>Lower Secondary School (LSS)</td>
<td>2 years (Classes VII – VIII)</td>
<td>Class VIII summative assessment</td>
</tr>
<tr>
<td>Middle Secondary School (MSS)</td>
<td>2 years (Classes IX – X)</td>
<td>Class X summative assessment</td>
</tr>
<tr>
<td>Higher Secondary School (HSS)</td>
<td>2 years (Classes XI – XII)</td>
<td>Class XII summative assessment</td>
</tr>
<tr>
<td>Tertiary Education (Degree College)</td>
<td>3 years (1st – 3rd Degree)</td>
<td></td>
</tr>
</tbody>
</table>
Prior to the introduction of the new curriculum, the Bhutan Certificate of Secondary Education (BCSE) and the Bhutan Higher Secondary Education Certificate (BHSEC) were associated with the Council for the Indian School Certificate Examinations (CISCE), New Delhi, India. Thus, for several decades Bhutanese students studied the Indian school curriculum and sat for Indian examinations. This meant that for many years, Bhutan did not make the school curriculum more appropriate to her own history and values (Thinley, 2013).

Besides a series of school assessments conducted by the respective teachers, there are nationwide examinations at the end of Classes 6, 8, 10 and 12. Of these four assessments, the first two (Classes 6 & 8) are conducted by the schools using uniform question papers provided by the central Board of Examinations. The main purpose of these two assessments is to empower teachers to internally evaluate their students’ achievement level at the end of Primary school (Class 6) and Lower Secondary school (Class 8). Likewise, assessments are conducted at the end of Secondary School (Class 10) and Higher Secondary School (Class 12), although these examinations are conducted and evaluated externally by the central Board of Examinations.

Until quite recently, the general public education system was based on a curriculum imported from India. Consequently, all teaching materials were those prescribed for Anglo-Indian schools, except for study of the national language Dzongkha. From the mid-1980s, the then Education Department began to reform the education system in accordance with national requirements and intentions stated in the Second Quarterly Policy Guidelines (Ministry of Education, 2007). Thereafter, the development of a relevant curriculum and curriculum materials for schools throughout the country began following this important policy change.

The first reform came with the introduction of the New Approach to Primary Education (NAPE). This project, which spanned all general subjects in the primary school, emphasised activity-based learning, shifted the focus from teacher-centeredness to child-centeredness, as well as from remoteness of content to familiarity of content (Dolkar, 1995). Hence, the curricular focus was upon making learning meaningful and relevant and providing scope for students to construct their own knowledge and not just absorb information (Ministry of Education Bhutan,
2009). However, the medium of instruction was still English, as discussed further in the following section.

2.2.1 Medium of instruction in English

As argued by Dorji (2005) there is a great deal of discussion about the quality of education offered in Bhutanese schools. One of the existing concerns is the competency of students in English. Unlike the case in many Asian countries, the medium of instruction in Bhutanese schools has been English since modern education was introduced in the country. Except for the learning of the national language (Dzongkha), all subjects were taught in English, including Mathematics (Verma & Dorji, 2014). Teaching Mathematics in English added another huge challenge for Bhutanese students. For instance, a child studying in one of the rural schools enrolled in the school for the first time is challenged with three main new languages: English, Dzongkha and mathematics. Dzongkha itself is treated like a foreign language, as students have their own local dialect. Learning mathematical language in English terms is challenging for Bhutanese students. Hence, to date, understanding and solving mathematical word problems in English is still considered a huge barrier in terms of teaching and learning of mathematics in Bhutanese schools.

Moreover, use of English language is also a problem for some teachers, as argued by Yahaya, Noor, Mokhtar, Rawian, Othman, Jusoff (2009) in their studies conducted in Malaysia on teaching of mathematics and science in English. These found that the majority (85.2%) of participating teachers face difficulties in explaining concepts in mathematics in English. As such, these findings indicated that using English as a medium of instruction is not only a problematic for the students but also for teachers teaching the subject. Bhutan’s history and culture are unique in the Asian continent, because its geography has allowed it to remain a homogenous nation. Hence, other Asian countries where curriculum reform in mathematics has been undertaken cannot be done easily in the context of this research. Nevertheless, the literature review will draw on examples from Indonesia, Hongkong, and Malaysia, which are more heterogeneous nations in respect to ethnicity, language and religious diversity.

The preceding paragraphs have provided a brief introduction to the historical development of education in Bhutan. In the following sub-sections, more specific
details regarding the history, reform and content of mathematics education in Bhutan are presented.

2.2.2 Indian based Mathematics Curriculum

By 1959, there were approximately 20 government schools in the country. Curriculum and textbooks were borrowed from India, from where most teachers were recruited (Rinchhen, 2013). Bhutan was neither in a position to develop her own curriculum nor to write textbooks relevant to the Bhutanese context. As such, there was no mathematics textbook developed exclusively for Bhutanese children (McLean & Hiddleston, 2003). Examples cited in the textbooks and narrated by teachers were often far from children’s life experiences and their imagination (Subba, 2006). For instance, textbook examples referred to railway stations, trains, and ships, which were all unfamiliar to the typical Bhutanese child. In addition to inappropriate contexts, the Indian curriculum was criticised for its disjointed development of concepts and the misalignment of content with students’ ability (McLean & Hiddleston, 2003). These criticisms of the Indian-based curriculum have been the basis for reform over the past few decades.

The formal and comprehensive introduction of mathematics education in Bhutanese schools came with the start of modern education in the early 1960s under the leadership of the third King. During these years, mathematics was normally presented in a very formal and abstract manner, modelling Indian practices and taught by Indian teachers. A more modern approach, associated with constructivist views of the teaching and learning of mathematics, was seldom practised. There was little opportunity provided for learners to construct their own knowledge by being actively involved in the teaching and learning process. Hence most of the time, the teaching and learning of mathematics tended to be procedural and resulted in instrumental understanding in which students learnt rules but lacked a deep conceptual or relational understanding. The impact of this was clearly shown in the results of the National Board of Examinations, although no formal research has been conducted suggesting reasons for the low performance of students. This deficiency of conceptual knowledge was evidenced every year when very few people opted to study mathematics at universities and colleges (Dolma, 2002).

As seen in the findings of the Education Sector Review Commission (2008), the outcome of mathematics education in Bhutan produced an average of only 35%
of Class 12 students scoring high enough for admission to tertiary institutes. Another study, the *National Education Assessment*, also indicated the poor performance of students in mathematics (Ministry of Education, 2003). This poor performance has affected the supply of skilled people in important fields like medicine, industry and technology, as well as in mathematics and science teaching.

Like its earlier paper in 2003, the Ministry of Education paper of 2009 revealed that Bhutan has suffered from the poor performance of its school children in mathematics (Curriculum and Professional Support Division, 2008; Ministry of Education Bhutan, 2009; Royal Education Council & Educational Initiatives, 2010). It has taken a long time for Bhutan to realise the importance of learning mathematics in a globalised environment. As stated earlier, expatriate mathematics teachers, mainly Indian and with little understanding of the Bhutanese context, were hired to address the shortage of trained teachers (McLean & Hiddleston, 2003). A graduate from one of the Bhutanese colleges of education reflected on his experience of learning mathematics at school and stated that “teachers taught us through dictation, the only pedagogical approach teachers seemed to possess at that time”. He noted that “there was no variety or alternative ways, no examples related to our experiences outside the textbooks, we never learnt mathematics outside the classrooms” (Subba, 2006, p. 21). For example, to find the circumference of the circle, the students were given a ready-made formula ‘C = 2 π r’ and were not encouraged to think of ‘why does C = 2 π r?’

Further, distribution of concepts was age-inappropriate, resulting in concepts and skills being required of students before they were cognitively ready (Curriculum and Professional Support Division, 2005). For instance, complex and abstract mathematical topics like formal Algebra were included in the primary mathematics curriculum. Moreover, conceptual gaps within and between most class levels resulted in substantial leaps from concept to concept, which created gaps in foundational ideas that underpin the development of advanced mathematical concepts (Curriculum and Professional Support Division, 2005). In addition, the syllabus was conceptually difficult and complex for teachers to cover in one academic year. Teachers were left with no alternative but to rush through the ‘content’ superficially. In the process, they were unable to teach effectively, and children failed to gain a deep understanding of
mathematical concepts. Owing to this, there was a high motivation to reform the mathematics curriculum and prepare students to compete in the workplace.

2.3 **BHUTANESE MATHEMATICS EDUCATION REFORM**

Government policy is to prepare today’s students with skills which vary significantly from those required in earlier times. To do this, the Bhutanese government has looked to other countries’ mathematics and curriculum practices to inform reform. In North America, it has become necessary for today’s learners to reason mathematically, communicate intellectual ideas and solve problems innovatively (National Council of Teachers of Mathematics, 1989, 2000). According to the National Council of Teachers of Mathematics (2000, p. 21), “students’ understanding of mathematical ideas can be built throughout their school years if they are actively engaged in tasks and experiences designed to deepen and connect their knowledge”.

Hence, engaging students more productively during class sessions has become crucial in terms of helping students understand mathematics deeply and meaningfully (Perrin, 2008). The end result of students being able to understand mathematics is expected to bring some enjoyment in learning mathematics, which could ultimately lead them to experience GNH in mathematics classrooms.

As in North America, Bhutanese reform related to the teaching and learning of mathematics has placed great significance on nurturing students’ conceptual understanding of mathematics as well as building their problem-solving skills. Such ideas are described in the North American study conducted by Perrin (2008). Moreover, such reforms are associated with the constructivist view, focussing on students being able to build an internal and personal understanding to construct new knowledge (Chang & Fisher, 2003). According to the National Council of Teachers of Mathematics (2000), teachers are expected to design learning activities that are authentic and meaningful for children to construct their own mathematical knowledge. These are the types of learning activities in which students are encouraged to examine, connect, represent, communicate and solve mathematical problems. Helping learners to become mathematically capable employees is necessary to meet the demands of daily life and the ever-changing job market (Matthews, 2000). Such ideas of curriculum reform have influenced Bhutan.
In Bhutan, reform to improve the standards of mathematics education started gradually in the late 1980s. The new thinking was mainly based on questioning the relevance, needs and aspirations of the Bhutanese education system (McLean & Hiddleston, 2003). In the early 1990s, experts from universities in more advanced countries, mainly from the United Kingdom, were invited to share their ideas with Bhutanese mathematics educators, in particular the lecturers in Bhutan’s two Education Colleges. In the process, several sessions of professional learning activities were conducted for mathematics lecturers at the two Colleges, which provided lecturers with some exposure regarding how to deliver mathematics lessons more effectively.

In addition, within the years 1990 – 1994, several groups of lecturers from Bhutan’s teacher-training Colleges were sent to the University of Leeds to pursue a one year Advanced Diploma in Educational Studies in various disciplines, including mathematics. The author of this study happened to be one of those fortunate lecturers. The knowledge and skills gained from their study helped these lecturers to develop more effective teaching skills and strategies, and enabled them to contribute more in their respective Colleges. Later, more Bhutanese mathematics educators were sent abroad to universities in Canada and Australia to gain further exposure. Hence, over the last 20 years, Bhutan has drawn upon international practices to inform the localisation of its education system, both by employing its own Bhutanese teachers and by replacing the previous Indian curriculum with a Bhutanese curriculum for English and Mathematics (Fricot, 2009).

### 2.3.1 The new Bhutanese mathematics curriculum

In response to the 2003 National Educational Assessment, the Ministry of Education decided to undertake a review of the mathematics curriculum drawing upon recommendation provided by independent external advisors. This began in 2003, when two European consultants (i.e., United Kingdom), Dr. McLean and Dr. Hiddleston, performed a thorough study of Mathematics Education in Bhutanese schools. They were the first group of scholars who made strong recommendations for an immediate review and change of the mathematics curriculum (Ministry of Education Bhutan, 2009). Based upon these recommendations, in 2005 a team of consultants from Canada, along with their Bhutanese counterparts, began developing
the mathematics curriculum framework from Class PP to 12 (Ministry of Education, 2007).

As discussed earlier in this chapter, four main limitations were identified in regard to the past mathematics syllabus (Curriculum and Professional Support Division, 2005):

- Conceptual gaps;
- Distribution of concepts;
- Delivered by Indian teachers; and
- Taught in English.

The conceptual gaps were in terms of establishing foundational ideas in students, which has been addressed in the current curriculum through gradual construction of concepts over the years, allowing students time to assimilate ideas before moving on to more complex ideas. Similarly, the second gap was addressed by re-distributing mathematical concepts to align with currently accepted practices and standards in other developed countries, as well as with needs specific to Bhutan. Finally, as discussed earlier, the third and fourth gaps were mathematics lessons delivered in English and by foreigners, mainly from India.

The mathematical concepts outlined in the new curriculum represented a sequential and logical developmental framework for mathematics instruction. All changes (additions and deletions) to the current curriculum have been guided by standards set by the National Council of Teachers of Mathematics (1989; 1991; 1995; 2000). Many of the mathematics curriculum reforms beyond Bhutan have been based on or informed by these standards. Further, the intention of the new curriculum was in line with the intentions set by the Ministry of Education (MoE) in the document *Purpose of School Education for Bhutanese Schools PP-VIII* (Ministry of Education, 2002). That document clearly set the goal of:

> Providing mathematics learning experiences through frequent opportunities to explore, discover, describe mathematical patterns and relationships, and extend mathematical experiences by involving students in problem solving in small group interactions (2002, p. 25).
Furthermore, the new mathematics curriculum was designed to reflect the findings of research around the world aimed at improving the standard of teaching and learning of mathematics. The same idea was re-emphasised by Lyonpo Thinley Gyamtsho, then the Minister of Education in his Foreward address in support of the reformed mathematics curriculum (Curriculum and Professional Support Division, 2005), in which he stated that:

Mathematics is important not only in the advancement of science and our understanding of the workings of the universe, but it is also equally important to individuals for personal development, and in the work place. It prepares the learners with a powerful set of tools to understand and change the world. These tools include logical reasoning, problem-solving skills, communication skills and the ability to think in abstract ways. Hence, mathematics is important in all spheres of life. (p. 3).

Accordingly, the following key features were drawn from the NCTM standards (National Council of Teachers of Mathematics, 2000) and integrated in the new mathematics curriculum framework (Curriculum and Professional Support Division, 2008), with more emphasis on a constructivist view of teaching and learning of mathematics in the classroom, where students are:

- encouraged to explore the given task on their own, before the teacher’s explanation;
- required to justify the strategies used and adopted;
- encouraged to explain their own learning and thinking using various forms of representation (e.g., enactive, iconic, symbolic or in written/verbal modes);
- to use technology/concrete objects/authentic materials in the mathematics classroom to provide hands-on experiences.

As listed above, the new curriculum builds on the reforms initiated in the NAPE (New Approach to Primary Education) period which was briefly discussed in an earlier in Section 2.2 of this chapter. The features of the new curriculum were intended to strongly support a child-centred approach to the teaching and learning of mathematics. The new curriculum was intended to promote teaching mathematics
through an activity-based approach encouraging students to be active both mentally and physically.

Such ideas were associated with studies conducted by Gokcek (2009, p. 1052), who argued the need “to create a learning environment where students are mentally and physically active”. Adopting such ideas, the Curriculum and Professional Support Division (2005) developed a policy in regard to providing professional support for mathematics teachers in having hands-on experiences themselves before they actually implement the reformed ideas in the classroom. Teachers with such experience were then expected to provide learning tasks which help their students gain abilities in problem-solving, reasoning, communicating, representing and making connections as presented in Figure 2.1.

![Figure 2.1. The birth of curriculum intentions](image)

The synthesis of constructivist theory, child-centred and activity-based practices, and the aims of conceptual understanding and the development of problem-solving ability were summarised in the new curriculum as four intentions (Curriculum and Professional Support Division, 2008, pp. 9-10). They are:
Intention 1: Greater emphasis given to the need of the students to understand mathematics rather than memorise rote procedures.

Intention 2: Emphasis more on why something is true and not simply that it is true (e.g. $2 \times 3 = 6$, why? How should students understand this?)

Intention 3: Use of contexts that are meaningful to the students (they can be either mathematical contexts or real world contexts)

Intention 4: Encourage more students in using process standards of ‘communication’, ‘reasoning’, ‘making connections’, and ‘representation’ and help students to develop problem-solving skills.

The content of the new curriculum was based on extending learners’ thinking. The relevant mathematical concepts were intended to be well-connected and coherent within and between mathematical contexts. The examples cited in the curriculum documents were mostly based on what children experience outside the classroom as in Intention 3. For instance, while learning the concepts of fractions, one context refers to the cooking of curry (ema-datchi), a common Bhutanese meal. In addition to this, the teachers were expected to encourage students frequently in reasoning, how? and in relation to why? more than simply what?, as stated in Intention 2. In this sense, students were to be provided with tasks where they had the opportunity to express and justify their understanding through various forms of representation (i.e., verbally, iconically and symbolically). Through such contexts, students would be provided with opportunities to apply some of the integrated process standards, such as reasoning, connection, communication and representation and so Intention 4 is achieved. In this way, students are encouraged to understand the concept and apply mathematics more meaningfully in their daily activities outside the classroom, such as sharing and measuring as intended by Intention 1.

A teacher having considered all of the intentions while planning a lesson was expected to encourage the students to construct their own knowledge by applying all the process standards while completing a task. Thus, learners would be motivated to solve the given mathematical task more meaningfully. These curriculum intentions characterised a new mathematics curriculum framework that represented a major
departure from the previous curriculum. Besides the content adjustment and contextualisation of concepts, the new mathematics curriculum focuses on conceptual knowledge and relational understanding (Ministry of Education, 2007).

Respecting the fact that changing practice is difficult, the introduction of the new curriculum was accompanied by materials such as textbooks for the students and guidebooks for teachers. These materials were intended to allow and encourage teachers to align their teaching practice with the NCTM standards and principles. Each class level was provided with these materials, which were developed based on the respective curriculum framework in the order of sequence. For instance, textbooks were developed based on the content included in the framework for each class level. Similarly, the Teacher’s Guidebook was written to help teachers implement the suggested ideas presented in the corresponding Student Textbook. Like any other standards-based curriculum (Goldsmith & Mark, 1999), this new textbook included materials for teachers to have students work in small groups discussing mathematics, explaining their reasoning, and coming up with multiple approaches to solving a problem. Moreover, each lesson in the textbook includes a sample of a learning task in the form of a Try This activity. These two resources were introduced to help teachers understand and then implement the new curriculum effectively. The layout of the textbook illustrates the approach of activity-based learning, as every topic has a series of learning activities, as shown in Appendix C. The only task left for the teachers was to modify some of the given activities according to the local context to facilitate students’ learning. Unlike some of the manuals or guidebooks written in the past, this particular guidebook was produced with the intention of helping teachers to use the new textbook.

The guidebooks also contained suggested approaches to formative assessment that a teacher could use while conducting lessons. Provision of a Teacher’s Guidebook was based upon the assumption that the teacher had exposure to the underlying principles that shaped the curriculum design. In Figures 2.2 – 2.6, excerpts from both the Guidebook and the Textbook are presented to illustrate the books’ content. The excerpts are taken from the Class 5 curriculum in a unit that explores the topic of fractions. This unit follows the structure that is common to all units in the curriculum, in which each unit is divided into three sections: Getting Started; Regular Lessons; and Unit Revision. This unit structure is reflected in both
the Teacher Guidebook and the Student Textbook. In addition, the Teacher Guidebook provides information regarding the mathematics of the topics, the rationale for the approach taken, and guides to both formative and summative assessment.

The *Getting Started* section is about finding the readiness level of the students for learning the specific mathematical topics included in the unit. Normally, it is intended to be conducted using authentic learning activities provided to the students prior to the introduction of the specific topic. For instance, in the unit on fractional numbers, the Getting Started activity integrated all concepts of the unit and involves students actively revealing their understanding of fractions based on their experience and knowledge gained from their previous class levels (PP – 4).

The *Regular Lessons* section detailed the suggested teaching and learning approach for each topic. According to the new curriculum, a whole lesson is expected to be taught using an instructional design comprising four components, as illustrated in Figure 2.2, aligning with the intentions of the new curriculum (Curriculum and Professional Support Division, 2008). The order of the sequence may not be always the same but it is expected a lesson should possibly contain all these, particularly the first component (exploration).

![Figure 2.2. Suggested lesson components](image)

The four lesson components were suggested in the new curriculum for teachers to help students gain the relational type of mathematical understanding. *Exploration* is included to explore students’ existing knowledge through thought provoking learning activities. Based on the level of students’ knowledge identified from exploration, a teacher is expected to conduct *exposition* whenever students are in
need of the teacher’s support. The conduct of formative assessment is expected to take place along with teaching, particularly before moving on to the next concept, and use problems/tasks from practice and applying these to enhance understanding of the taught concepts. This type of practice is based on a constructivist view of teaching mathematics through the adoption of NCTM principles and standards (National Council of Teachers of Mathematics, 2000), where teaching is expected to take place based on students’ readiness for learning. Hence, if all these suggested components are followed consistently, the curriculum is designed to help students’ gain relational understanding and thus derive deep conceptual knowledge. In the following paragraphs the four components are illustrated using a sample topic taken from the Class 5 fractions unit.

**Exploration phase**

The textbook accompanying the new mathematics curriculum advocated that lessons introducing a new concept should begin with an activity referred to as Try This. The proposed nature of the Try This activity was intended to be meaningful and realistic to students. Moreover, the content of the activity was intended to provide opportunities for students to apply process standards intended in the new curriculum. The example of a Try This activity shown in Figure 2.3 is intended to encourage the students to connect with their previous knowledge. Moreover, the activity is based on the children’s experiences outside the classroom. The importance of realistic problems is viewed as relevant and appealing to students as they relate to their daily experiences (Perrin, 2008). The tasks are intended to provide learners with ways to communicate their thinking processes by sharing and interacting with peers and teachers. Hence, teachers are expected to design similar learning activities, particularly when introducing a new topic or concept.
Figure 2.3. Suggested strategies from textbook and guidebook.

The nature of the Try This activity shown in Figure 2.3 is based on the use of contexts that are meaningful to the student, so it is intended to encourage the students to make connections between the mathematical concepts (i.e., the mixed number \(1\frac{1}{4}\) and the improper fraction \(\frac{5}{4}\)), and communicate mathematically (i.e., either express the given problem verbally or write symbolically) and be able to justify their constructed knowledge (i.e., of ‘5 times in quarters’), requiring stirring the soup and so achieve Intention 2. To scaffold the students’ implementation of the process standards and the achievement of the curriculum’s objectives, the Teacher Guidebook provides specific guidance, including questions to ask the students, as indicated in Figure 2.3.

**Exposition Phase**

Based on the diagnostic assessment conducted during the exploration phase, the teacher should develop an exposition suited to the students’ specific needs. The Teacher Guidebook provides some suggestions as to what the exposition might include, and accompanying notes are included in the Student Textbook. Examples of both are presented in Figure 2.4.
As discussed with reference to the Exposition phase, teachers are provided with guidance to help students use the suggested ideas in the textbooks. In the formative assessment phase, the depth of the students’ understanding of the taught lesson is assessed. An example is shown in Figure 2.5. In that example, students are expected to relate the main ideas presented during the exposition to the preceding Try This activity. During this phase of the lesson, the objective of the planned lesson is said to be achieved if the majority of students are able to positively respond to the formative assessment activity. In the case that the formative assessments suggest limited understanding by the majority of students, the Teacher Guidebook provides suggestions of follow-up exposition that may be useful in further developing the
students’ understanding, as presented in the first part of Figure 2.5. Such further guidance is shown in the Student Textbook.

Students’ Textbook (p. 80)

Teachers’ Guidebook (p. 130)

Figure 2.5. Examples of formative assessment

Practising and Applying Phase

Once the majority of students demonstrate sound understanding, the lesson then moves onto the Practising and Applying phase. Excerpts from the Teachers’ Guidebook and Students’ Textbook are presented in Figure 2.6.
As shown in Figure 2.6, the problems/tasks suggested in this lesson component were intended to provide opportunities for students to apply all the process standards. For instance, in Question 1 (Student Textbook), students are encouraged to think and interpret the iconic mode of representation in symbolic form (e.g., $\frac{3}{4}$) to enhance their understanding on the taught topic ‘renaming fraction as division’. Likewise, in Question 4 students are encouraged to relate and apply understanding of the concept to solve their day-to-day problems experienced outside the classroom. In
this way, all the process standards adopted in the new mathematics curriculum are integrated for the students to apply. Further, a small section under *Unit Revision* is suggested in the textbook to assess students’ understanding of the taught mathematical concept in summative form, for the purpose of recording students’ performance on the unit.

### 2.3.2 Roll-out of the new curriculum

This section describes the process by which the new curriculum was introduced, including the conduct of professional learning activities to support the curriculum’s phase-wise commencement. Although development of the new curriculum framework for classes PP – 12 began in 2003, actual classroom implementation began only from 2008. Additionally, commencement of the new mathematics curriculum for each class level could not be started simultaneously due to delays in writing both the Student Textbooks and Teacher Guidebooks. As such, the commencement of the new curriculum took place phase-wise, starting from Classes 9 and 10 from the top and Classes 4 and 5 from the middle. The main reason for leaving out Classes 11 and 12 was due to their affiliation with the Delhi Board of Examinations and the lack of textbooks. In 2015, the Classes 11 and 12 Teacher’s Guidebooks had been written but the textbooks were still being published. The phase-wise introduction of the new curriculum is summarised in Figure 2.7, in which the shaded portion represents the curriculum of the respective class levels being taught.

<table>
<thead>
<tr>
<th>Class</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP</td>
<td>2008</td>
</tr>
<tr>
<td>I</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td></td>
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<tr>
<td>V</td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td></td>
</tr>
<tr>
<td>VII</td>
<td></td>
</tr>
<tr>
<td>VIII</td>
<td></td>
</tr>
<tr>
<td>IX</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td></td>
</tr>
<tr>
<td>XI</td>
<td></td>
</tr>
<tr>
<td>XII</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 2.7. Phase-wise introduction of the new mathematics curriculum.*
Prior to the actual commencement of the new curriculum, several workshops were intended to be conducted with mathematics teachers. These workshops formed what was referred to as the Training of Trainers (ToT) program. They were workshops particularly focused on the applications of standards and principles adopted by the new mathematics curriculum. However, the workshops took place only once and could be categorised as ‘one-stop’ workshops.

As depicted in Figure 2.7, the first ToT workshop was organised at the national level and facilitated by the team of experts (including consultants from abroad), who were actually involved in developing and writing curriculum documents including the curriculum framework, the Student Textbooks, and Teacher Guidebooks. This national level workshop, held in 2007, was attended by a selected group of mathematics teachers from across the country who represented the country’s six different regions. Following this, it was intended that the attendees would conduct training at the regional level, attended by teachers representing the schools in the region. These teachers were then meant to train the teachers at their respective schools. Unfortunately, the regional ToT workshops took place in only a few education regions, primarily because of budgetary constraints.

Although the effectiveness of the ToT program may have been compromised, there was support for the development of teachers. The Ministry of Education believed that high quality teaching is the most important factor for success of the
education system (Ministry of Education, 2013b), and that there is an imperative to update teachers on new developments in curriculum and other educational issues through in-service training and workshops organised at national, district and schools levels. However, as with the ToT program, it has proven difficult for the government to offer professional learning activities to teachers as frequently as required. Key reasons for this have been budget constraints and geography.

To date, teachers have been paid to attend professional learning activities which have required them to come from different corners of the country. Some travel far on foot taking days to attend the workshop, in which case the government must pay for travel and basic minimum daily allowances. As such, to conduct any type of workshops is very expensive, particularly when trying to cover a large number of participants. For these reasons, it is highly likely that not all Bhutanese teachers have received the necessary training for successful implementation of the new curriculum. Hence, there is a high chance that the majority of Bhutanese teachers have faced difficulties in integrating the ideas and standards intended in the new curriculum because they have been ill-prepared in regard to the underlying principles upon which the curriculum is based.

2.3.3 Summary

As discussed earlier, with the introduction of modern education in the country from the early 1960s, Bhutanese school children have been learning mathematics through a foreign language, English. Mathematics itself is a language of thought, and to communicate that in English has indeed been a very challenging task for Bhutanese school children and teachers. Moreover, for several decades, in most of the schools in Bhutan mathematics was taught by teachers recruited mainly from India, who came from a different cultural and social background.

Consequently, over the past decades school children in Bhutan have faced numerous challenges in learning mathematics. For instance, children solving mathematics problems related to their life experience beyond the classroom were rare. Most of the time, children were instructed from the blackboard using unfamiliar examples cited from the textbook and based on the Indian context and culture. There was little chance for them to develop strategies other than mechanically following rote procedures. However, from 2008, with the introduction of the new curriculum, the learning environment in the classroom was expected to improve. But, due to the
lack of timely professional learning activities in relation to the intentions and standards of the new curriculum, this study anticipates that there was and remains a significant misalignment between the reform-oriented intentions of the new curriculum and the realities of classroom practice.

2.4 BHUTANESE PRE-SERVICE TEACHER TRAINING PROGRAMS

This section describes the development of pre-service teacher-training programs in Bhutan. Coinciding with the introduction of public schooling in the 1960s, there has been a series of pre-service teacher education programs. Table 2.3 provides a summary of these programs, the details of which are presented in the following sub-sections.

Table 2.3

Summary of Bhutanese Teacher Education Programs

<table>
<thead>
<tr>
<th>Program</th>
<th>Year of offer</th>
<th>Pre-requisite</th>
<th>Program duration (years)</th>
<th>Certification</th>
<th>Annual enrolment (approx.)</th>
<th>Status / Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Teaching Certificate</td>
<td>1968</td>
<td>Class VIII</td>
<td>2</td>
<td>Year PP – Year 6</td>
<td>10 – 15</td>
<td>Up-graded to PTC</td>
</tr>
<tr>
<td>2. Primary Teacher Certificate</td>
<td>1981</td>
<td>Class X</td>
<td>2</td>
<td>Year PP – Year 6</td>
<td>50 - 100</td>
<td>Phased out in 2002</td>
</tr>
<tr>
<td>3. Bachelor of Education (Secondary)</td>
<td>1983</td>
<td>Class XII</td>
<td>4</td>
<td>Year 7 – Year 10</td>
<td>30 - 40</td>
<td>Current</td>
</tr>
<tr>
<td>4. Bachelor of Education (Primary)</td>
<td>1993</td>
<td>Class XII</td>
<td>3</td>
<td>Year PP – Year 10</td>
<td>50 - 100</td>
<td>Phased out in 2008</td>
</tr>
<tr>
<td>5. Post-graduate Diploma in Education</td>
<td>2006</td>
<td>Bachelor Degree</td>
<td>1</td>
<td>Year 7 – Year 10</td>
<td>20 - 30</td>
<td>Current</td>
</tr>
<tr>
<td>6. Bachelor of Education (Primary Curriculum)</td>
<td>2008</td>
<td>Class XII</td>
<td>4</td>
<td>Year PP – Year 6</td>
<td>100 - 150</td>
<td>Current</td>
</tr>
</tbody>
</table>

2.4.1 Teaching Certificate for Primary School Teachers

The program known as the Teaching Certificate for Primary Teachers was introduced in 1968, along with the launch of the first Teacher Training Institute (TTI) in the country (National Institute of Education, 1985). At that time, there was no system in place for students to study beyond Class 10 within the country. Most of the students who enrolled in this program had the academic qualification of Class 8 (Year 8) certificate. According to Dukpa (2013), there were two groups of student teachers at TTI: the first comprised students who came directly from schools, while the other
group comprised students who were already teaching in schools. The second group were typically less qualified but were already working as temporary teachers because of the shortage of teachers. Based on their performance in schools and their interest in pursuing a teaching career, those temporary teachers were given a chance to obtain the Teaching Certificate issued by the Ministry of Education.

This two year teaching certificate program prepared students to teach general subjects in primary schools. Those graduates were posted to schools with minimum qualifications for teaching primary school children. Being the first training program, the course structure was undeveloped and could offer only basic teaching skills and strategies. Moreover, there was no established staffing: the program was taught by a handful of lecturers, mainly from India. Often, expert teachers were invited during their vacations to lecture on some of the issues related to teaching skills and strategies. The first candidates graduated in 1970 and the program continued for almost a decade. This was the first time Bhutan produced her own formally qualified primary school teachers, although in very small numbers. Despite low graduate numbers, the nation was proud of producing her own certified primary teachers for over a decade, solving some of the problems in regard to the teacher shortage. This program was upgraded to the Primary Teacher Certificate, as outlined in the following section.

2.4.2 Primary Teacher Certificate

In 1981, the Teaching Certificate program was replaced by the Primary Teacher Certificate (PTC) program. The PTC program accepted high-school graduates with a Class 10 Certificate (National Institute of Education, 1985, 1988). Like the previous program, the PTC had a duration of two academic years. However, this program was more refined and richer in its course structure. For instance, in mathematics, the course structure was divided into two parts, mathematical content (40%) and teaching methodology (60%). The PTC program, which lasted less than three decades and was phased out in 2002.

In the PTC program, students were trained to teach all primary subjects, including mathematics. The program was composed of four specific strands to cater for the four core subjects of the primary school curriculum: mathematics, English, science and social studies. Each strand comprised two modules corresponding to content, one each for lower and upper primary. The first year of the program was to
prepare student teachers to teach lower primary level students (Classes PP – 3). The second year of the program included topics that were more related to upper primary school students, such as system of numbers, fractions, percent, ratio and proportion and geometry. Associated with those modules were two learning objectives related to constructivist views of teaching mathematics (National Institute of Education, 1985):

1. Create interest in mathematics among students by organising appropriate activities;

2. Use mathematical information from the child’s daily life and other areas of the curriculum.

The two constructivist-aligned objectives introduced the PTC pre-service teachers to the activity-based approach, albeit in very small measure. The design of the two mathematics modules was influenced by an activity-based approach to teaching mathematics. For instance, while introducing the concepts of ‘fractions’, student teachers were exposed to several teaching and learning aids such as the fraction board, concrete objects, coloured counters and the number line, which they were expected to incorporate into their assignments. Hence, student teachers would have some ideas regarding the type of teaching and learning of mathematics that should take place in school classrooms.

The use of an activity-based approach to mathematics teaching and learning was a result of the direct influence of mathematics educators who had been sent abroad for further professional studies. Moreover, the international academics who trained the Bhutanese educators were also able to review the two Primary Teacher Certificate mathematics curriculum modules. Later, those two modules were further strengthened as a result of a long partnership program between the Bhutanese Teacher Training Colleges and Zurich Teacher Training College in Switzerland. In this way, the seeds of activity-based learning were sown in two Bhutanese colleges of education in the early 1990s.

In the Primary Teacher Certificate program, the focus was more on using games and concrete objects as teaching and learning materials to motivate the learning of mathematical concepts. However, there was no inclusion of any learning theories to provide the background to this new child-centred approach. Thus, despite the reformed mathematics curriculum, the majority of Primary Teacher Certificate
teachers might be expected to use teacher-centred practices because they lacked exposure to theoretical principles during their training period.

2.4.3 Bachelor of Education (Secondary)

In 1983, Bhutan’s first Teacher Training Institute was upgraded to become the National Institute of Education, which coincided with the introduction of a new program, the Bachelor of Education (Secondary). This program is still offered by the country’s two Colleges of Education. The entry requirement for this program is Class 12. A graduate from this course is qualified to teach students from Classes 7 to 10 in their chosen subject area (National Institute of Education, 1997). This program is offered for all core subjects (i.e., Mathematics, English, Physics, Chemistry, Biology, Geography, Economics and History). The course comprises a wide variety of modules that focus on personal and professional development and specialisation in any two specific subjects of the students’ choice depending on their stream (i.e., Science, Arts or Commerce). The duration of this program has been increased from three to four years.

With regards to the training of mathematics teachers, the program’s course structure is made up of two elements: professional training in the philosophy and practice of mathematics teaching, and specialisation in mathematics content. As such, the course structure is divided into ten modules (seven for content and three for methodology), which are closely integrated with the major concepts of the school mathematics curriculum. This includes time allocated to the careful study of the texts available for use in secondary school classrooms, including the newly created Teachers’ Guidebooks and Students’ Textbooks. The content of the program was intended to reflect the needs of Bhutan and Bhutanese society.

For quite some time, the number of students opting for this program in the subject area of mathematics remained low, with only one or two students each year. One of the main reasons for this low enrolment could be due to lack of interest in joining the program or students not meeting the entry requirements for the program. Moreover, the content of the course is almost the same as the pure mathematics offered in other degree colleges (e.g., science). This has made it very difficult for the Colleges’ mathematics faculty to spare any time for pedagogical development.
2.4.4 Bachelor of Education (Primary)

In 1993, the Bachelor of Education (Secondary) program was complemented by the introduction of the Bachelor of Education (Primary) program. The main difference between the Bachelor of Education (Primary) program and the Primary Teacher Certificate program was that the Bachelor students majored in one specific subject in which they were qualified to teach up to Class 10. They were also qualified to teach all other core subjects at the primary school level. This was a three-year program (Verma & Dorzi, 2014). A unique feature of the program was that it placed equal emphasis on the learning of theory and of practice. Attempts were made to insert the most recent philosophies of learning and best practices into all of the program modules. The modules were developed and arranged to provide a good balance between theory, practice and connectivity with the primary school curricula.

Besides content modules, there were three modules offered on pedagogy, including one module purely for selected learning theories on mathematics education. In this way, all students in this program were prepared to teach mathematics in primary school. The Bachelor of Education (Primary) was phased out from 2008 and is being replaced by the Bachelor of Education (Primary Curriculum). A brief outline of this new program is presented in Section 2.4.6.

2.4.5 Postgraduate Diploma in Education

In 2006, the Postgraduate Diploma in Education was made available. This program is offered to candidates with a Bachelor’s degree who are trained for one academic year. Typically the graduates are placed in higher secondary and tertiary teaching positions. Most become lecturers in colleges under the Royal University of Bhutan. The graduates from a mathematics related degree course such as commerce and science were encouraged to choose mathematics as their teaching subjects in the schools, but are mostly found to be working in tertiary colleges and institutes.

2.4.6 Bachelor of Education (Primary Curriculum)

In 2009, a four-year Bachelor of Education (Primary Curriculum) program replaced the three year Bachelor program. Currently, it is the only available teacher education course for primary school teaching. It is a combination of the Primary Teaching Certificate and the Bachelor of Primary Education, and prepares graduates to teach in all subject areas up to Class 6 level. The program was developed to closely align
with the intentions of the new mathematics curriculum. From 2009, the two teacher-training colleges were merged with the Royal University of Bhutan, and were no longer under the Ministry of Education. This period has also seen an increase in the standards students need to graduate, which is in turn reflected in the standard of the content in each of the program’s modules (Verma & Dorzi, 2014).

Owing to all these changes, particularly in mathematics, the program focuses on a balanced combination of theory and teaching practice. Thus, the nature of the program is quite different from the previous PTC program. Moreover, this program is presented with much clearer specific aims and objectives. A total of 40 modules are offered and four modules are allotted specifically for teaching of mathematics. Of the four, one is specifically designed as an ‘Introduction to Mathematics Education’ to familiarise student teachers with current learning theories related to mathematics education.

In addition to the theoretical content, the basic ideas associated with NCTM process standards and principles are also included, a move that was initiated in 2003 when this NCTM-related content was introduced into the Bachelor of Education (Primary) and Bachelor of Education (Secondary) programs. This was done mainly to exemplify some of the learning theories related to the constructivist approach for the teaching and learning of mathematics.

2.4.7 Summary
As described in the previous sub-sections, six different pre-service teacher training programs have been introduced in Bhutan over the past decades, of which three remain. In the evolution of these programs, more emphasis has been placed on the development of primary education programs, so as to foster a high quality foundation for Bhutanese school children. The most recent primary education program, the Bachelor of Education (Primary Curriculum), has been developed to incorporate international standards in terms of both quality and qualification.

2.5 THE LANDSCAPE OF EDUCATION IN BHUTAN
Paralleling changes to the programs for pre-service teacher education, the teaching environment in Bhutanese primary schools has changed significantly in terms of student enrolment and school infrastructure. As described by Rinchen (2013) *Reflections on the Education Journey*, when reflecting upon his own schooling in
Bhutan, there were “limited rooms in the school and there was no proper accommodation, the same room was used as classroom during the day and dormitory at night” (p. 12). Similarly, the memories of a school student in the 1950s included “during my time, one could easily count the number of schools and teachers on fingertips, but now due to increase in the enrolment of students both in urban and rural areas, it is extremely difficult to accommodate them” (Lhamu, 2013, p. 3).

Since the early years, there have been major changes to the landscape of education in Bhutan. Currently, there are around 20 to 50 schools in each district and each school enrols between 100 – 200 students every year. Moreover, instead of government officials visiting families to convince parents to send their children to schools, parents rush to admit their children to school. As a result, the current number of students in the classrooms is approximately 36 – 40, particularly in urban schools. Consequently, the total number of students sitting the national examination for Year 10 has also increased annually, as shown in Table 2.4.

<table>
<thead>
<tr>
<th>Year</th>
<th>Boys</th>
<th>Girls</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>24833</td>
<td>26766</td>
<td>51599</td>
</tr>
<tr>
<td>2012</td>
<td>24530</td>
<td>26298</td>
<td>50828</td>
</tr>
<tr>
<td>2011</td>
<td>23606</td>
<td>25228</td>
<td>48834</td>
</tr>
<tr>
<td>2010</td>
<td>22958</td>
<td>23764</td>
<td>46722</td>
</tr>
<tr>
<td>2009</td>
<td>21627</td>
<td>21978</td>
<td>43605</td>
</tr>
<tr>
<td>2008</td>
<td>20353</td>
<td>20502</td>
<td>40855</td>
</tr>
</tbody>
</table>

Source: Ministry of Education (2013a)

Over the years, the balance of gender has also shifted and in recent years there is parity. This shift to gender balance is also reflected in the number of males and females who graduate from Bhutanese education colleges. In 2013, approximately 40% of graduates were female, whereas in early years the numbers of female graduates were few. Not only is the number of females enrolling in each Education degree increasing, the total number of female teachers graduating from Bhutan’s colleges is also increasing.

Despite the increasing number of graduates from the two Colleges of Education, there remains a shortage of teachers. Consequently, every graduate is
easily absorbed into the education system. As per the current policy of the Ministry of Education, the best graduates are placed in remote schools so that they can be rewarded by the Ministry of Education (MoE) faster and have the chance to advance their career. This type of incentive is treated as a motivation for teachers to serve in remote schools, because they bear many hardships not endured by their fellow teachers in urban schools. Thus, when considering the profile of experience across the schools, it is fair to say that recent graduates are more often found in remote schools and senior teachers in urban schools, as will be demonstrated in Chapter 5.

The system of placing new graduates in rural or remote schools provides a rough picture in terms of teachers’ qualification, age and their teaching experience across the country. With the phasing out of two previous programs (PTC & Bachelor of Education (Primary) in the early 2000s, graduates of these programs are more likely to be found in urban and semi-urban schools. Moreover, the average age and teaching experience of such teachers could range from 30 – 40 years and they will have had 10 – 20 years of classroom experience. This will be further discussed in Chapter 5.

2.6 CHAPTER SUMMARY

The aim of the Bhutanese education system is to ensure that practices keep abreast of those in more advanced countries. However, sometimes these intentions have been constrained in their implementation. A constraint is the slow response of the system in terms to the introduction of the new ideas. Most ideas are diluted in the process and resources are wasted: the Bhutanese government has invested millions of dollars in hiring experts and consultants from abroad to share their latest ideas to help update the prevailing education system.

On a personal note, for nearly two decades of service rendered as a mathematics lecturer in one of the premier colleges of education in Bhutan, there was never a system where the newly updated ideas were evaluated in the field. Teacher graduates were never evaluated on skills and strategies learnt during their training period. This reflects a tendency to be negligent in terms of applying both ‘pressure and support’ by the system to new graduates to encourage their professional development as they enter the field. Consequently, graduates easily fall back on the prevailing practices of senior teachers in the schools, practices with which they are
familiar and have been previously subjected to. In the process, the sharing and subsequent implementations of new ideas tend to be quite limited and ultimately of little benefit for school students. Similarly, due to the limited round of workshops to orientate teachers to the ideas of the new mathematics curriculum, it was quite unrealistic to expect that the intentions of the new curriculum would be implemented successfully by classroom teachers. Hence, the findings of this study could raise awareness of any issues regarding the alignment between curriculum intentions and actual classroom practices, thus providing an impetus for improving the system.
This chapter reviews literature in three main areas pertinent to identifying and understanding the alignment between the intentions of the new curriculum and the actualities of classroom practice. First, literature regarding teachers’ beliefs and their impact on the teaching of mathematics is investigated. Second, literature regarding the theory and practices of social constructivist-based mathematics education is reviewed. To illustrate social constructivist practices, approaches from the USA’s NCTM standards and the Dutch Real Mathematics Education (RME) approach are employed. Third, literature regarding the challenges of various standards-based curriculum reforms faced across the world, particularly in countries with similarities to Bhutan, is appraised. Together, these three major sections provide a theoretical foundation upon which to understand the nature of the Bhutanese curriculum’s intentions and expectations for classroom practice. The conceptual framework drawn from this literature is then situated within the Bhutanese context, and an initial framework for analysing practices in relation to the intentions of the new mathematics curriculum is presented. The literature review concludes by restating the overarching research aim and the Research Questions in regard to the relevant literature.

3.1 BELIEFS

A mathematics teacher’s practice in the classroom is said to be influenced by the beliefs they have about mathematics and mathematics education (Ambrose, 2004). This section of the chapter explores the notion of teachers’ beliefs and how these might affect the implementation of the new mathematics curriculum in the Bhutanese school context. In addition, a short description of how teachers’ beliefs can impact on students’ own beliefs about mathematics is presented.

3.1.1 The nature of beliefs

The concept of beliefs is considered to be very complex and has no fixed definition. Zimmerman (2006, p. 1), stated that “belief cannot be defined”. Beliefs have been described as personal principles (Tarmizi, Tarmizi, & Bin Mokhtar, 2010) or internal
schema (Goldin, 2002) developed and constructed from the accumulation of experiences which an individual encounters in life and unconsciously aligns with new information to guide action. Beliefs are relatively stable, but changes to beliefs take place when a person is deeply affected by being repeatedly exposed to new experiences that contradict their existing beliefs, and that are believed to help individuals make sense of a changing world and influence how new information is recognised or excluded.

Charalambous, Panaoura, and Philippou (2009) have argued that beliefs are related to personal understandings grounded on ideas about the world. Beliefs are taken as the individual’s dispositions toward actions and serve as filters to complex situations, thereby making them comprehensible and shaping the individual’s interpretation of events (Ambrose, Clement, Philipp, & Chauvot, 2004). For instance, when a person is faced with challenging situations, it is their beliefs that compel them to act in particular ways.

Borg (2001) presented his comparative view distinguishing between beliefs and knowledge. According to Borg (2001, p. 186), beliefs are argued as “a mental state” which one holds and accepts as true, “although the individual may recognise that alternative beliefs may be held by others”, whereas “knowledge must actually be true in some external sense”. Similarly, Pajares (1992, p. 313) argued that “belief is based on evaluation and judgment; knowledge is based on objective fact”. For instance, as argued by Ernest (1989), two teachers may have the same knowledge, but teach in different ways because of the powerful effect of beliefs on their decision making.

Roehrig and Kruse (2005) argued that a belief is a personal construct important to the holder’s practice. However, Leatham (2006) pointed out that beliefs cannot be unswervingly perceived or measured but must be inferred from what people say, intend and do. As concluded by Lazim, Abu, and Wan (2004, p. 2), “beliefs play a significant role in directing human’s perceptions and behaviour”. Therefore, the nurturing of beliefs in a person is important so that they can make appropriate decisions while facing challenges (Ambrose et al., 2004). Hence, findings from past studies tend to indicate that one’s behaviour and conduct are said to be the direct result of one’s beliefs, which could be applicable to teachers and their conduct depending on their beliefs, as discussed in the next section.
3.1.2 Types of teachers’ beliefs

As reported by several studies, teachers’ beliefs have a significant impact on their teaching practices (Guskey, 1985; Richardson, 1996; Tobin & McRobbie, 1996). For instance, teachers with the highest levels of reformed based teaching practices exhibited the most reformed beliefs (Roehrig & Kruse, 2005). Aligning with the preceding section on the nature of beliefs, this section presents the types of beliefs possessed by teachers in terms of teaching and learning approach. Teachers’ beliefs are supposed to have been instituted in the teacher’s own individual value system which, in turn, has been shaped and reinforced through personal experience as a student, student teacher in formal teacher training, teaching experience and family upbringing (Raymond, 1997; Van Driel, Beijaard, & Verloop, 2001; Walls, Nardi, von Minden, & Hoffman, 2002). At the same time, it has been found to be very challenging for teachers to match their behaviour with the type of beliefs they hold (Leatham, 2006).

Keys (2005) outlined three different beliefs that teachers in general are likely to possess:

1. Expressed beliefs: Beliefs and understandings which teachers express in words or text;
2. Entrenched beliefs: Internalised knowledge and beliefs built on experience but difficult to express;
3. Manifested/Enacted beliefs: Beliefs and knowledge that are evident in the practices and actions of the knower.

Teachers’ conduct in the classroom is the reflection of the third type (manifested), which could be influenced mainly by the second type of beliefs (entrenched). According to Keys’s findings, expressed beliefs function as idealistic views not fully embraced, but nevertheless expressed by the teacher to provide verbal support to beliefs that are enacted but not shown in practice. Thereby, according to Leatham (2006), teachers’ belief systems cannot be fixed through a process of substituting certain beliefs with more appropriate beliefs. Reasonably, teachers’ beliefs must be confronted in such a way that projected beliefs are seen by teachers as the most workable beliefs to adhere to.
3.1.3 The influence of teachers’ beliefs on mathematics education practices

As discussed earlier in Section 3.1, because of their idiosyncratic nature there can be no single statement or definition about mathematical beliefs to which individuals can refer (McLeod & McLeod, 2002). However, several studies have been conducted where the influence of beliefs on mathematics education practices has been identified. Some of these studies are presented in this section.

Cooney (2001, pp. 18-19), after conducting five studies on teacher change, concluded that “clearly teachers’ conceptions about mathematics and mathematics teaching strongly influence if not dictate their movement toward a reform-oriented teaching environment”. However, Sowder, Philipp, Armstrong, and Schappelle (1998) noted contextual hindrances to teacher change, including lack of organisational support and school resources, contextual factors which shape teachers’ practice in the classroom. Hence, it is important for researchers to understand the historical context of teachers’ experience in their school setting before making claims of any gap between beliefs and their practices (Herbel-Eisenmann, Lubienski, & Id-Deen, 2006).

Leder and Forgasz (2002), in a study of teachers’ mathematical beliefs and their impact on students’ learning of mathematics, found a strong influential relationship between beliefs and cognition. These authors argued that both cognitive and affective factors such as beliefs must be explored if an understanding of the nature of mathematics learning is to be improved. As such, their findings were closely aligned with mathematics education research which has focused more on cognition and far less on affect. Such research reflects the beliefs of people who considered mathematics to be a purely intellectual endeavour in which emotion plays no important role (Goldin, 2002).

Raymond’s (1997) model, which is summarised in Figure 3.1, suggests a strong relationship between teachers’ beliefs and their classroom teaching practices. It also suggests relationships between how teacher’s beliefs are shaped and developed.
Figure 3.1. Raymond's (1997) model of beliefs and practices.

Looking at the figure, teachers’ past experiences seem to have impact on the type of beliefs developed in teachers. Beside this, the type of teacher education program and teaching practice (Hollingsworth, 1999) tend to have some influence as well in shaping teacher’s beliefs. Likewise, a teacher’s teaching practices are also influenced by the immediate classroom situation and the prevailing social teaching norm in the school.

Several research findings have revealed that teachers’ choices regarding classroom practices, including planning and instruction, are closely linked to their knowledge and beliefs concerning mathematics and teaching (Ambrose, 2004; Hill, Schilling, & Ball, 2004). Beliefs play a significant role in a teacher’s selection and prioritisation of goals and actions; and teacher’s beliefs are likely to become apparent when there is a shift in goals during teaching (Aguirre & Speer, 2000; Mansour, 2013). For example, Aguirre and Speer (2000) presented a detailed analysis of how teacher beliefs interact with goals and influence moment to moment actions, through case studies of two secondary mathematics teachers teaching algebra. The results tend to reveal that particular collections of beliefs become apparent when there is a shift in the teacher’s goals. As Irez (2007, p. 17) argued, “beliefs are the best indicators of teachers’ planning, decision-making and subsequent classroom
behaviour”. Hence, several researchers (Chapman, 2002; Ernest, 1997; Liljedahl, Rösken, & Rolka, 2006; Stipek, Givvin, Salmon, & MacGyvers, 2001) have concluded that general beliefs about teaching have a profound effect on decisions related to practice.

Aligned with this idea, Vace and Bright (1999) argued that to bring about an effective change in classroom practice, teachers need to be encouraged to reflect on their beliefs and the impacts that their beliefs have upon their teaching. It is very unlikely that teachers can permanently modify their teaching practices without first having considered their values and beliefs (Beswick, 2006a; Handal & Herrington, 2003; Yu, 2008). This point is strongly supported by Wilkins (2008), who argued that the development of teachers’ own understanding of their beliefs is considered an important step for teachers in understanding their own instructional practices and ultimately in shaping those practices. Therefore, the development of sound instructional practices in the mathematics classroom is said to be possible only if more stress is placed upon the development of teacher beliefs (Wilkins & Brand, 2004).

Further, Seaman, Szydlik, Szydlik, and Bean (2005, p. 197) argued that “teachers’ beliefs about subject matter and about the nature of teaching indicate something about the culture of the educational system that produced them”. To instigate change in the classroom, it is said to be important to change what occurs in the teacher’s mind (Selcuk, 2004). Apart from knowledge of the subject and its teaching, researchers emphasise that it is very important that the teacher’s beliefs about the subject are addressed (Charalambous et al., 2009) since their teaching approaches are the direct result of their personal beliefs regarding the nature of mathematics (Handal & Herrington, 2003). Aydin, Baki, Kogce, and Yildiz (2009) also argued that the way teachers conduct their lessons depends on their beliefs about the subject. Hence, it is critical that mathematics teaching reforms reflect deeper changes in the system of beliefs held by mathematics teachers: Ernest (1989) stated that “teaching reforms cannot take place unless teachers’ deeply held beliefs about mathematics and its teaching and learning change” (p. 249).

Ernest (1989), from a philosophical perspective, proposed that there are three views in relation to mathematical beliefs prevalent among mathematic teachers: instrumentalist, Platonist and experimentalist. The instrumentalist views
mathematics as a bag of tools and believes that learning mathematics is the accumulation of facts, rules and skills. The Platonist views mathematics as a static but unified body of certain knowledge that is discovered but not created by learners. Finally, the experimentalist (or problem solver) views mathematics as a dynamic and continually expanding field of human creation and invention. Instrumentalists are more content and skills focussed, whereas experimentalists take a more explorative focus. Instrumentalist and Platonist beliefs consider mathematics as a subject of rules and formulas to be memorised, whereas experimentalists see mathematics as an interesting, beautiful, enjoyable and dynamic subject to be explored, invented and re-discovered. These three philosophical views provide a useful framework to consider mathematics education practices.

A range of criticisms have been made of teachers of mathematics who adopt approaches based on instrumentalist beliefs. First, approaches that are often described as traditional tend not to use hands-on materials and interesting things about the world outside the mathematics classroom (Warren & Nisbet, 2000). Beliefs and practices of instrumentalist teachers are often associated with their own experience of traditional school mathematics programs (Seaman et al., 2005, p. 5):

Most teachers have experienced traditional school mathematics programs and in their minds, mathematics largely consists of meaningless memorization of mathematical facts, rules, and procedures and they see their role as delivering such procedures.

Second, instrumentalists do not foster student autonomy but maintain a social context in which mistakes are something to be avoided rather than an opportunity for learning and the development of understanding (Stipek et al., 2001). Third, teachers with instrumentalist beliefs tend not to see the importance of posing problems and providing opportunities for students to explore problems on their own and to discover the intended knowledge themselves (Suurtamm & Vezina, 2010). Thus, an instrumentalist teacher plays the role of instructor with a focus and focus on content and encourages students towards the passive reception of knowledge.

In contrast with instrumentalists, an experimentalist teacher’s successful use of student-centred strategies is based on an awareness of the originality of students’ thinking (Wilson & Cooney, 2002). Experimentalists tend to present diverse and authentic tasks to the students and are more aligned with the constructivist model of
learning (Beswick, 2007; Warren & Nisbet, 2000). The experimentalist teacher plays the role of a facilitator by focusing more on learners in terms of guiding their exploration and construction of knowledge (Ernest, 1989). These teachers believe that students learn mathematics most effectively when they construct meaning themselves rather than being told by the teachers (Wilson & Cooney, 2002). For the experimentalist, mathematics is treated not as something one can transmit but as something that needs to be constructed by the students through authentic contexts and by using appropriate models and representations. These teachers attempt to establish a mathematics classroom where students are actively engaged in learning with deep understanding through exploration and cooperative work (Lloyd, 1999).

According to Beswick (2006a), the beliefs held by teachers regarding teaching and learning are said to be based on the teachers’ understanding of the purpose of mathematics. In particular, experimentalists consider that mathematics helps people to make sense of things around them and as a way of thinking to assist with difficulties faced in our daily lives (Mohamed & Waheed, 2011). Beswick’s (2006) work implies it is very important for teachers to understand the nature of mathematics and the way mathematics is used in everyday life. Hence, these scholars suggest that the learning environment provided should enhance learning mathematics in a realistic, dynamic and enjoyable manner, so that children are able to make sense of what is learnt and gain a deeper understanding of the concepts.

Like the experimentalists, Platonists emphasise understanding but through the teacher’s explanation, rather than reliance upon students’ autonomous construction of knowledge (Ernest, 1989). Like the instrumentalists, Platonists also believe mathematics to be a static body to be discovered but not created (McLeod, 1992). For them, mathematics is treated as a unified body of knowledge isolated from other domains of knowledge. This indicates that Platonists tend to fall in between the instrumentalists and experimentalists in terms of their beliefs and classroom practices.

From a Buddhist cultural perspective, the teaching and learning of mathematics in Bhutan has been influenced by practices related to instrumentalist beliefs. In Buddhist monasteries, prior to teachers’ explanation of scripts, monks are asked to memorise volumes of scriptures without understanding a word. Similarly, students in traditional Bhutanese classrooms had to memorise all the rules and formulas of the
respective concept before they were immediately asked to use it after the demonstration of problem solving (Subba, 2006). This current study aims to explore and understand the beliefs that Bhutanese mathematics teachers have about mathematics and mathematics education and how these beliefs affect the implementation of the new mathematics curriculum and ultimately affect students’ learning. The following section further describes the impact of teachers beliefs on students’ beliefs and performance.

3.1.4 Impact of teachers’ beliefs on students’ beliefs and performance

As stated in the literature (e.g., Peterson, Fennema, & Carpenter, 1989) there is a strong relationship between beliefs of the mathematics teacher and students’ understanding and achievement in mathematics. There is general agreement that students’ beliefs about mathematics develop gradually through being immersed in the culture of the mathematics classroom set by teachers, as argued by Verschaffel, Greer, and De Corte (1999).

As previously discussed, Wilkins (2008) argued that developing teachers’ understanding of their own beliefs is an important first step for teachers in understanding and ultimately re-shaping their pedagogy which will in turn influence their students’ achievements and beliefs. Hence, it is unreasonable to attempt to change the practice of teachers without also considering changing their beliefs (Beswick, 2006a). The consensus amongst a number of researchers (Kloosterman, 2002; Leder & Forgasz, 2002; Lester, 2002) is that research on teachers’ beliefs is essential because teachers’ beliefs can have a substantial impact on their students’ beliefs. Using Ernest (1989) classification of beliefs, the relationships between teachers’ beliefs and their associated classroom practices are summarised in Table 3.1. These beliefs and their associated practices are discussed and compared in the paragraph that follows.
Table 3.1

*Influence of Teachers’ Beliefs upon the Mathematics Classroom*

<table>
<thead>
<tr>
<th>Beliefs about the nature of mathematics</th>
<th>Beliefs about mathematics Teaching</th>
<th>Beliefs about mathematics learning</th>
<th>Role of the teacher</th>
<th>Intended outcome in students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instrumentalist</td>
<td>Content-focused with an emphasis on performance</td>
<td>Skill mastery, passive reception of knowledge</td>
<td>Instructor</td>
<td>Skills mastery with correct performance</td>
</tr>
<tr>
<td>Platonist</td>
<td>Content-focused with an emphasis on understanding</td>
<td>Active construction of understanding</td>
<td>Explainer</td>
<td>Conceptual understanding with unified knowledge</td>
</tr>
<tr>
<td>Experimentalist</td>
<td>Learner-focused</td>
<td>Autonomous exploration of own interests</td>
<td>Facilitator</td>
<td>Confident problem posing and solving</td>
</tr>
</tbody>
</table>

Source: Ernest (1989)

A teacher’s role in the classroom is determined by the teacher’s belief about mathematics and mathematics education, with consequences for outcomes in students. Ernest (1989) has argued that beliefs about mathematics and mathematics education play a very important role in helping to shape students’ beliefs about the nature of mathematics. If a teacher is experimentalist, this ultimately helps to develop a belief in the students that the learning of mathematics is interesting and enjoyable, solving tasks as they make sense of them and bringing mathematics closer to real problems in life (Lowrie, 2004). Therefore, the type of role played by the mathematics teacher with regards to mathematics is said to be very important in shaping learners’ beliefs about the nature of mathematics.

There is increasing evidence that students’ beliefs about mathematics shape their understanding of mathematical concepts and their performance on mathematical tasks (Tsamir & Tirosh, 2002). This was evident in the work of Peterson et al. (1989) who investigated the relationship between first grade teachers’ pedagogical content knowledge and students’ achievement in mathematics. Peterson et al. (1989) found a strong correlation and significantly positive relationships between teachers’ beliefs, teachers’ knowledge, and students’ achievement in mathematics. Others have gone further by claiming that changes in teaching practice can directly influence students’ attitudes: “changes in practice entailed changes in students’ beliefs about the nature of mathematics as well as pedagogy” (Presmeg, 2002, p. 294). Such changed practices provide students with opportunities to expand their beliefs about
mathematics education (Aydin, Baki, Yildiz, & Kogce, 2010). To achieve this, teachers are to consider both sociological and emotional perspectives as they establish a supportive and interactive climate in their classrooms, set up social norms and select tasks for students to engage with (Lester, 2002). Thus, the affective climate of the classroom can influence students’ attitude to mathematics, and in turn lead to higher performance in the discipline. Therefore, the beliefs of teachers are considered as an important aspect of this thesis.

### 3.1.5 Summary of teachers’ beliefs

In brief, teachers’ beliefs are considered to play a very important role in mathematics education reform, which has underpinned the significant research conducted in recent decades (Leatham, 2006; Munirah Ghazali & Sinnakaudan, 2014; Speer, 2008). As argued by Wilkins (2008), developing teachers’ understanding of their own beliefs is an important first step for teachers in understanding and ultimately re-shaping their pedagogy, which will in turn influence their students’ achievements. It is unreasonable to attempt changing the practice of teachers without also considering changing their beliefs Beswick (2006). Since the new Bhutanese curriculum is based on constructivist theories, its effective implementation depends on teachers having beliefs aligned with these philosophical foundations. The following section will explore the constructivist philosophical foundation of the curriculum.

### 3.2 THEORY AND PRACTICES OF SOCIAL CONSTRUCTIVIST MATHEMATICS EDUCATION

In Chapter 2 the underpinning principles of the new Bhutanese mathematics curriculum were described. The new curriculum aimed to be student-centred, activity-based and to incorporate aspects of the students’ outside-school life. That is, the new curriculum was aligned to constructivism. In this section, a brief overview of constructivist theory provides a basis for the analysis of mathematics education practice. To illustrate how this theory can be applied in practice, aspects of the American NCTM Standards and the Dutch Realistic Mathematics Education program (RME) are summarised. The NCTM’s standards were selected because of the direct influence they had upon the new mathematics curriculum in Bhutan. The rationale for choosing the RME approach was its similarity to NCTM standards and its direct relevance for classroom practice. Consideration of these two approaches to social constructivist-based mathematics education provides a basis for proposing an initial
analytical framework with which to explore Bhutanese teachers’ mathematics classroom practices.

3.2.1 **Overview of constructivist theory**

Constructivism is a theory about how people learn and it has implication for teaching in the process of learning. There seems to be a connection between learning as a constructive activity and teaching in ways that acknowledge constructivism as the way in which students learn (Von Glaserfeld, 1990). Since this study is about exploring the implementation of the new constructivist inspired curriculum, it is necessary to first explore constructivist theory in more detail. As summarised by Powell and Kalina (2011), there are two main types of constructivist practices that have been adopted in the classroom: (1) cognitive or individual constructivism, based upon Piaget's theory, and (2) social constructivism, based upon Vygotsky's theory. Although others exist, they have not been as influential in framing mathematics curricula or pedagogy.

Similarities between these forms of constructivism include investigation-based teaching methods and students’ construction of concepts built on existing knowledge via contexts that are relevant and meaningful. Further, current mathematics education reforms tend to be geared towards the constructivist view of learning by doing, which involves using the learners’ previous knowledge in their current real-life context. Differences in these forms of constructivism include language development theory: in cognitive constructivism, thinking leads to the development of language, whereas in social constructivism, language leads to the development of thinking (Powell & Kalina, 2011).

In a related way, in cognitive constructivist theory, Powell and Kalina suggest ideas are constructed by individuals through a personal course of action, whereas in social constructivism, ideas are constructed through communication (i.e., social interaction) between the teacher and other students in the classroom. Further, according to Derry (1999), social constructivism highlights the importance of culture and context in understanding what occurs in society, and constructing knowledge based on this understanding. This perspective is closely linked with many contemporary theories, most notably the developmental theories of Vygotsky and Bruner and Bandura's social cognitive theory (Schunk, 2000). Through this constructive process of learning, students eventually construct the intended
knowledge rather than rehearsing it until the examination is completed and then forgotten (Brooks & Brooks, 1999). Moreover, according to Von Glaserfeld (1990), learning is said to be dependent upon the learner’s existing knowledge, which itself has been constructed through earlier experience.

Lowrie and Logan (2006), in their study of the use of realistic contexts to foster mathematical thinking, conducted an investigation on the influence that a genuine artefact has on students’ spatial reasoning. They found that students are more likely to utilise a range of spatial skills to complete mathematics tasks when they are deeply engaged in an activity. In accordance with constructivist views of learning, mathematics is intended to be learnt through active involvement of students (Hurst, 2011). The teacher’s role is said to be one of facilitating students’ learning through the provision of authentic learning activities, expected to arouse and motivate learners through the provision of authentic materials related to real situations (Bickmore-Brand, 1998a).

These principles were articulated in the new Bhutanese curriculum. A key goal of curricula based upon the social constructivist view is to achieve learning that can be described as engaging, thoughtful and meaningful to the students. Hence, as intended in the new Bhutanese curriculum, students would be expected and encouraged to use prior knowledge to create or construct new knowledge in response to further experience. In the following section, a brief discussion that explores and describes the implications of social constructivism in mathematics education is presented.

3.2.2 Social constructivism in mathematics education

Constructivist theory has the potential to influence the role of the mathematics teacher and students as well as the pedagogical approaches used in the classroom. As pointed out by Brown and Coles (2012), in their classroom-based studies of how expert teachers reflect on their teaching, all learning is doing and all doing is learning and ultimately learning is equivalent to action. Teachers are no longer considered the only authority for learning in the classroom. Rather, students are encouraged to construct their own mathematical knowledge rather than receiving it in fixed form from the teacher or a textbook (Perry, Geoghegan, Owens, & Howe, 1995). According to Brooks and Brooks (1999), a constructivist approach is the key to building a deep mathematics are understanding in students.
According to constructivists, relational understanding of mathematics is considered to occur through active engagement of students in both cognitive and physical aspects. In support of this proposition, Hadi (2002), in a study of teacher professional learning activities relating to the introduction of a new approach in Indonesia (based upon RME), presented findings which revealed that *doing mathematics* was rated as the best approach by participants. The result of Hadi’s study implied that learning takes place only when the learners are involved in doing something on their own in a relevant context and using authentic learning tools. It is through learning by doing that learners are engaged both cognitively and physically, and are expected to make sense of the concept, ultimately leading towards deeper understanding of mathematics.

Further, Goldsmith and Mark (1999), in their discussion about the purpose of standards-based mathematics curriculum in the United States, have referred to the term constructivism as “students being actively involved in building their own understanding” (p. 40). Aligning with this definition, these authors supported the argument that curriculum must enable students to make sense of mathematics and at the same time recognise and value their own mathematical thinking. Students are expected to derive knowledge through collaborative investigations and hands-on explorations using various representations and discussion (Goldsmith & Mark, 1999). Hence, social constructivism serves as a basis for many current reforms, including those in mathematics education such as NCTM (connection to students’ daily experience) and RME (horizontal connection). Such student centred learning in mathematics will be more thoroughly discussed in Section 3.2.3.

Moreover, teachers and administrators are expected to be in a position to support students by providing learning materials that will promote a rigorous and constructivist based mathematical environment for them to develop both skills and deep understanding. Extending this point, teachers’ deep and flexible understanding of mathematical concepts could help in providing richer learning opportunities for students. The implication is that mathematics learning requires the learning experience to engage students actively with appropriate resources supported by knowledgeable teachers. The focus of learning has shifted, as McLean and Hiddleston (2003) argue, from product to process. Constructivist environments are claimed to provide this opportunity since central to constructivist theory is the
recognition of the influences of prior knowledge and experience upon learning. Thus, in mathematics education, the implication is that the content of mathematical activities should be based on children’s life experiences so that they can find solving mathematical problems both easier and more enjoyable.

House (2006) conducted a study on relationships between instructional activities and the mathematics achievement of adolescent students in Japan. This study demonstrated that students in classes where the teacher frequently included situations from everyday life for solving mathematics problems tended to have higher mathematics test scores. Moreover, Gutstein (2006), in his book on *Reading and writing with mathematics*, argued that students should be provided with situations matching their interests and the content they are required to study. Other researchers (Ball & Bass, 2000; Perry et al., 1995; Rogers, 1999) have emphasised the importance of engaging learners actively in a learning world that is relevant to them. This supports the claims of Rogers (1999), who suggested that providing students with such learning environments could encourage students to develop an appreciation for mathematics, experience the joy of mathematics and provide opportunities to explore its beauty.

Ball and Bass (2000), in a literature review focused on the construction of mathematical knowledge in the elementary classroom, have argued that learners should be provided with situations in which they can construct relevant mathematics themselves. A similar point was made by Perry et al. (1995) in their report on cooperative learning and social constructivism in mathematics education: students attributed much of their success in their mathematical development to a supportive environment in which they cooperated. Therefore, as described by Smith (1999), it is important for teachers to choose learning problems and situations that will actively involve students and stimulate student interest in how mathematics is applied to real-world situations.

Aligning with this point of providing a suitable situation for learning, there needs to be a shift in the role of the teacher from an explainer to a facilitator paying attention to all students, fulfilling the NCTM’s equity principle (Lingefjard & Meier, 2010). Further, to help students reach a targeted learning level, teachers are expected to design an appropriate learning activity, from which students’ ability levels could be identified (National Council of Teachers of Mathematics, 2000). In the process, a
teacher’s role could be best described as facilitator in guiding students to perform a learning activity. For instance, in a case study of two experienced teachers implementing the strategy of mathematical modelling as participants in the Comenius Network in Germany and Sweden, Lingefjard and Meier (2010) explored the role of the teacher as a manager of learning. In this role the teachers supported their students in the problem solving process without pushing them towards one specific solution. They posed diagnostic questions to stimulate student thinking and supported them to ultimately solve problems on their own. Hence, the preferred situation for learning should be one that requires learners to understand, explain, defend and evaluate (NCTM, 2000).

Wittmann (2005), in his analysis of the Realistic Mathematics Education initiative, has argued that mathematical knowledge is not expected to be transmitted top-down to be absorbed and digested. Rather, the social context of the classroom in which learning occurs should be considered. To contrast these principles with the traditional approach, a simple example of a contemporary approach is narrated here, based on the researcher’s own experience as a student. In the past, to teach a mathematical concept of fractions, a teacher could have started a lesson with the following definition: *a fraction is a number with denominator and numerator*. In the contemporary approach, children are guided to derive the true meaning of the mathematical concept and generate their own definition such as *a fraction is either a part of a whole object or a part of whole set, provided the parts are equally divided or parted*. Instead of defining the fraction in terms of two abstract and meaningless terms (numerator and denominator) introduced at the beginning of instruction, the students might be given concrete objects (e.g., a bar or a set of chocolates) with which they construct the meaning of fractions themselves through cutting (equal parts) and sorting (equal groups). Then, drawing on this experience, the more formal mathematics language might be introduced. Hence, the new approach is considered as more real and meaningful in introducing the concept in a physical form and drawing upon children’s previous knowledge.

Two approaches of learning mathematics frameworks, NCTM standards and Realistic Mathematics Education, are described next to help illustrate how constructivist principles inform curricula. These examples are useful benchmarks to examine the alignment of the new Bhutanese mathematics curriculum with
constructivism, and provide a model for the implementation of ideas from NCTM standards directly and Realistic Mathematics Education indirectly.

3.2.3 **Examples of social constructivist mathematics education**

In the following sub-sections, two examples of contemporary mathematics education practice are presented to illustrate the application of social constructivist theory. In the sense, these two approaches were considered to promote and provide materials to create a rigorous and constructivist based mathematical learning environment in which students develop both skills and deep understanding (Goldsmith & Mark, 1999). These two approaches are then compared.

**Example 1: NCTM Standards**

The first example of a document describing contemporary mathematics education based on social constructivism is the standards and principles developed by the NCTM (1989; 1991; 1995; 2000). The NCTM standards, created by the world’s largest mathematics education organisation, has offered a vision of more effective teaching, learning and assessment of mathematics (K – 12). Teaching mathematics in the 21st century is considered to be one of most challenging tasks for both teachers and students and yet it is a must for students to learn the subject (NCTM, 2000):

> In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens door to productive futures. A lack of mathematical competence keeps those doors closed…all students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding (p. 80).

According to Mathematics (2009), learning mathematics is maximised when teachers emphasise mathematical thinking and reasoning. An emergent body of research studies on mathematics education tends to support NCTM’s position on school mathematics (Bosse, 2006; Fillingim, 2010). According to Perrin (2008), in North America, the NCTM, via its principles and standards, has led the call for reform for several decades. This reform places an emphasis on nurturing students’ deep conceptual understanding of mathematical strands as well as developing their problem solving skills (Goldsmith & Mark, 1999). Moreover, according to (Van de Walle, Karp, & Bay-Williams, 2013), teaching in the high achieving countries
(measured through TIMSS) bears greater resemblance to the endorsements of the NCTM standards than does teaching in USA.

Despite drawbacks argued, the findings from several studies have indicated a positive achievement in students’ learning mathematics using standards based curriculum (Wiske & Levinson, 1993). For instance, Riordan and Noyce (2001) have conducted studies on the impact of standards-based mathematics curriculum on student achievement in Massachusetts for both middle and elementary school students. The results from both the schools were significantly better on the 1999 state-wide mathematics test compared to students in the traditional program. Such findings helped the authors to conclude confidently that a standards based mathematics curriculum has a positive influence on student achievement. The findings also indicated the depth of positive influence of standards based curriculum on students, which was observed consistently all across all students irrespective of gender, race and economic status.

Similarly, Wood and Sellers (1997) conducted a study on comparison of students’ achievement between problem-centred instruction (standards based) and use of a more traditional textbook in class. The findings indicated that students from the problem centred instruction group not only performed better in standardised tests in Grade 1 through 4, but also demonstrated greater conceptual understanding in numeration, place value and multiplication. Students able to master these three basic mathematical concepts are likely to be advantaged learning higher-level mathematics later. The same point was supported by Boaler (1998), who found in her study that students from a traditional school were much less able to apply their mathematical knowledge to real situations than students from a reform oriented school.

The NCTM standards encourage teachers to create a classroom culture in which students are actively involved in learning through discussing and discovering mathematics. Moreover, pedagogical practices associated with the NCTM standards have been found to have a strong influence on improving student performance in mathematics (Alba, 2001; Fillingim, 2010; Matthews, 2000). In addition, examples used in mathematics classrooms are proposed to be based on the children’s life experiences outside the classroom to help learners make sense of learning, thereby making learning mathematics more enjoyable and meaningful. However, traditional view holders argue that standards are taking mathematics in the wrong direction by
providing insufficient mathematical knowledge to teachers and lack of mastery of calculation by students (Loveless, 2015). For traditional view holders, students learn mathematics by observing teachers, presentation of ideas and through careful explanations followed by organised opportunities for students to connect with their previous experiences, to practice and to evaluate the learnt knowledge (Riordan & Noyce, 2001). Hence, as argued by Perrin (2008), making NCTM’s vision of school mathematics a reality requires many classroom teachers to modify their current practices.

In the following paragraphs, a brief summary of NCTM principles and process standards are presented. These are then related to the philosophical intentions and associated with practices of the new Bhutanese curriculum. The National Council of Teachers of Mathematics (2000) defined the six principles listed in Table 3.2 and claimed that they are fundamental to high-quality mathematics education. These principles constitute a vision to guide educators as they strive for the persistent improvement of mathematics education in classrooms, schools, and educational systems.

Table 3.2  
*NCTM Principles (2000)*

<table>
<thead>
<tr>
<th>Principles</th>
<th>Definition/explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>Excellence in mathematics education requires equity—high expectations and strong support for all students.</td>
</tr>
<tr>
<td>Curriculum</td>
<td>A curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well-articulated across the grades.</td>
</tr>
<tr>
<td>Teaching</td>
<td>Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.</td>
</tr>
<tr>
<td>Learning</td>
<td>Students must learn mathematics with understanding, actively building new knowledge from experience and previous knowledge.</td>
</tr>
<tr>
<td>Assessment</td>
<td>Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.</td>
</tr>
<tr>
<td>Technology</td>
<td>Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning.</td>
</tr>
</tbody>
</table>

When applying the aforementioned principles, a teacher is expected to consider and integrate them when planning mathematics lessons. Teachers are expected to adopt the principles in planning lessons for students while they perform learning tasks. In addition to the principles, the NCTM (2000) defined five process standards that describe ways students acquire and apply content knowledge, so enabling meaningful learning. The process standards as outlined in NCTM (2000) state that students should:

- become proficient in making connections between mathematical concepts and real-world applications;
- share their understanding of math with peers and their teacher;
- analyze information shared by peers and the teacher;
- use models, diagrams, and mathematical symbols in the process of learning mathematics;
- make informed decisions based upon evidence;
- make and analyse mathematical conjectures; and
- experience and asked to grapple with solving a problem and during such activity, the teacher should act as a facilitator of learning.

The NCTM (2000) claimed that when students are provided with opportunities to use these processes, learning is effective in promoting mathematical understanding, leading towards mathematically literate, informed and productive citizens. As argued by Lowrie (2004), when given opportunities to solve problems using authentic materials in realistic situations, students become capable of developing skills in knowing when and how to use mathematical knowledge in their practical life activities. In the following paragraphs, a few examples are presented describing both successful and unsuccessful attempts at implementation of NCTM based curricula.

Matthews (2000), in a doctoral study conducted in the US state of Delaware, examined mathematics teaching after the introduction of a constructivist mathematics curriculum aligned to state and to national NCTM standards. He found that the instructivist approach to mathematics teaching which existed prior to the introduction of the new curriculum was well entrenched. The two curricula stated above were
compared based on their orientation with the state standards: Matthews concluded the two curricula represented different epistemologies. Matthews’ study revealed the problems faced by the Delaware teachers, such as coping with the task of adopting and using pedagogy in implementing mathematics curricula that met both the NCTM curriculum standards for mathematics and the Delaware content standards. Despite these problems, Matthews (2000) claimed that, compared to the instructivist approach, the NCTM standards based curriculum supported a wide-ranging approach to problem solving. Further, the NCTM standards based curriculum provided opportunities for students to develop and apply a variety of strategies to discuss and solve problems based on everyday situations, and share their thinking in relation to problem solving with their teachers and their peers. At the seat of the problem faced by the teachers was their limited knowledge of how to implement the NCTM standards-based curriculum. Teachers had not been provided with information about the problems, challenges and opportunities associated with implementing the standards. Implied in Matthew’s study is the importance of preparing teachers for change when implementing new curricula.

Riordan and Noyce (2001) conducted a study on the impact of NCTM standards based curriculum upon elementary and middle school student achievement in Massachusetts. These authors reported that standards-based mathematics programs produced a significantly better result than did more traditional programs. The term standards-based often refers to reform-based curricula which have philosophical alignment with the NCTM standards (Bay, Reys, & Reys, 1999). According to Goldsmith and Mark (1999), a standards-based approach emphasises the development of conceptual understanding and reasoning, thereby putting into practice the constructivist view of involving students actively in building their own understanding. Further, the standards-based curriculum stresses non-routine mathematical explorations, in contrast to traditional mathematics curricula that emphasise strengthened computational adeptness and algorithmic abilities.

Another study was conducted by Fillingim (2010) on the impact of one year of standards-based instruction at the third grade level in a rural elementary school in the southern United States. The findings provided evidence of successful application of NCTM standards in the mathematics classroom. For instance, within the sessions, participants in the study were found to have demonstrated confidence and flexibility
in developing and using multiple written representations to model and solve mathematical problems. Such evidence tends to add to the potential of representation in building the mathematical foundations of elementary students. Moreover, participants’ use of pictures, physical models and words exposed conceptual understanding and misunderstanding not obvious in tasks that focussed exclusively on computation and procedure.

The essence of Fillingim’s (2010) study was engaging participants in learning mathematics through problem solving. The author provided a series of learning tasks, in which opportunities were offered to make connections within and beyond mathematical knowledge, apply different strategies, and incorporate written and physical models, and explaining reasoning. By doing this, the components of the NCTM process standards such as problem solving, communication, connections, representation, and reasoning and proof were able to be described and evaluated. Fillingim (2010) findings provide support for the use of NCTM standards to offer opportunities for students to become engaged in learning mathematics meaningfully and productively.

The third example of evidence to support the impact of using NCTM standards in teaching and learning mathematics is a study by Alba (2001). The purpose of Alba (2001) study related to the current study. It was, “to examine high school mathematics teachers’ beliefs and instructional practices in relation to NCTM standards and investigate how these beliefs impact on their classroom practice” (p. 3). The difference between this study and that of Alba is with the level of students chosen for the study: the current study has chosen primary level mathematics whereas Alba’s focus was on secondary mathematics. Alba’s study indicated that a relationship existed between teachers’ beliefs in mathematics and mathematics instruction, although not always consistently. However, the teachers in Alba’s study who had positive beliefs about NCTM standards were found to express the highest confidence in their use of standards and this was reflected in their classroom instruction.

Alba (2001) pointed out that several studies have been conducted with respect to NCTM standards, which have found that classroom practices in the top-performing countries were aligned with the pedagogical practices advocated in NCTM standards. As indicated in several studies, instruction that is aligned with the
philosophical underpinnings of the NCTM standards is more effective than traditional mathematical instruction (Alba, 2001; Fillingim, 2010; Perrin, 2008). With such findings in mind, it tend to support that pedagogical practices associated with the NCTM standards can have an impact on improving student performance in mathematics.

The fourth example of NCTM based curriculum was found in the study conducted by Perrin (2008), which involved beliefs and reported practices of seventh and eighth grade mathematics teachers. This study found that teachers’ beliefs and practices were aligned in such a way that they helped students to reason mathematically, communicate ideas and solve problems by being their guides, and that their students’ learning was expedited when compared to more traditional approaches. Perrin (2008, p. 2) argued “today, as never before, students must be mathematically literate”. Consistent with this statement, the NCTM (1989; 1991; 1995; 2000) stress nurturing students’ conceptual understanding of mathematics, as well as building their problem solving skills.

Unlike some other studies, Markward (1996) and Perrin (2008) argued that the NCTM has been leading the current call for reform for over two decades. Perrin (2008) study revealed that:

- teachers who were aware of NCTM’s standards had significantly higher beliefs in NCTM’s vision than those teachers who were not;
- there was no significant correlation between teachers’ beliefs scores and their reported level of use of NCTM aligned practices;
- teachers who most consistently agreed with NCTM’s vision tended to use NCTM aligned teaching methods more frequently than teachers who least consistently agreed with them.

The findings tend to indicate the importance of providing a thorough orientation on the contents of principles and standards of the NCTM and its vision to all mathematics teachers. Based on his findings, Perrin (2008) argued that it is important for teachers to have exposure to and awareness of NCTM and its vision, to enable them to align their beliefs towards its teaching methods and practices.

In summary, the NCTM standards and principles have been chosen as one of two practical examples of the constructivist view of teaching and learning of
mathematics, because the new Bhutanese curriculum intentions are primarily based on NCTM standards and principles. Of the four key intentions of the new mathematics curriculum, the first three are associated with the application of the six NCTM principles. The fourth intention relates to the application of the five NCTM process standards. The RME (Realistic Mathematics Education) philosophy in terms of teaching and learning of mathematics is another important approach adopted in schools across the world, including some developing countries such as Indonesia. Brief details of RME are presented in the next section.

Example 2: Realistic Mathematic Education (RME)

This section discusses a framework that assumes similar principles to the NCTM. Although this framework did not inform the Bhutanese reform process, the Realistic Mathematics Education (RME) model has been adopted in many countries. RME was initiated by the Dutch educator Hans Freudenthal over 50 years ago and has become very popular among mathematics teachers in that country. RME was developed in response to a perceived need to improve the quality of mathematics teaching in Dutch schools (Dickison, Eade, Gough, & Hough, 2010). This program has continued to be developed based upon the ideas shared by Freudenthal during the 1960s. It later became the Dutch answer to the world-wide imperative to reform the teaching and learning of mathematics (Heuvel-Panhuizen, 1998). Currently, the RME approach is used in countries including the United Kingdom, the United States of America, Germany, Spain, Portugal, Brazil, Japan, South Africa, Indonesia and Malaysia.

RME is based on three key concepts, originally proposed by Freudenthal, which have been described and extended by his colleagues (Gravemeijer, 2010). These three concepts are: guided reinvention; didactic phenomenology; and the use of mediating models. The concept of guided reinvention recognises mathematics as a construction of human activity which is used to explain the lived world. Consequently, RME stresses students’ activity in which they reconstruct mathematical ideas (in ways similar to those by which the ideas were originally constructed) under the guidance of teachers (Sembiring, Hadi, & Dolk, 2008). This contrasts to more mechanistic approaches to mathematics education in which mathematics is presented as a ready-made entity.
In general terms, phenomenology is the study of a concept in relation to the context in which it was created. In RME, didactic phenomenology describes instruction in which mathematics ideas are described in relation to the contexts in which they were created. That is, exemplary solution procedures form the basis for the construction of more formal mathematics. Finally, the concept of using mediating models relates to the various roles that models serve in aiding the learner’s construction of knowledge. Initially, models are contextually bound. Later in the learner’s conceptual development, these models explain problems (Streefland, 1985), and this way, models are used to bridge the gap between informal knowledge and formal mathematics.

As depicted in Figure 3.2, which was adapted from Gravemeijer (1994), a student is expected to have a deeper understanding of a mathematical concept if provided with an opportunity to connect both horizontally and vertically while solving a given learning activity or a task. The concepts of horizontal and vertical connections were derived from the ideas of mathematisation formulated by Freudenthal (2002): horizontal mathematisation comprises connection from the world of existence into the world of symbols (informal to formal), while vertical mathematisation means connecting within the world of symbols (formal to formal).

Figure 3.2. RME’s horizontal and vertical mathematisation.
As presented in Figure 3.2, horizontal connections are expected to develop when a mathematical task is based on students’ immediate life-based activities or something which they can imagine or can make sense of a problem. For example, a contextual problem is solved when each of the two friends receives a part of a chocolate bar. A term half (e.g. $\frac{1}{2}$) of a chocolate can be introduced linking from a part of a bar ($1 = \frac{1}{2} + \frac{1}{2}$).

Vertical connections (shown with vertical lines) are developed when a mathematical task is integrated with students’ previous or future mathematical knowledge, and is familiar to them while solving a problem (Barnes, 2005). Finally, the whole process can be connected vertically and explained in the form of an algorithm ($1 - \frac{1}{2} = \frac{1}{2}$) and in words (a bar of chocolate: when one whole is shared by two people and one half taken away to be left with another half), as shown in Figure 3.2. Hence, this sample task is mathematised both vertically (words to symbols) and horizontally (informal/context to formal/halves in mathematical terms).

Based upon these three key concepts of RME, the developers of RME have developed views on how mathematics should be taught and learned, which they have described as six principles (Van den Heuvel-Panhuizen, 2000). Some of the principles are more closely related to the actions of the teacher and some are more related to the student. The six RME principles are summarised as follows.

**Activity principle.** According to Freudenthal (1973), mathematics is an activity that can be best learned through doing. Students are treated as active participants in the learning process, in which they are encouraged to develop mathematical tools and understandings by themselves. Hence, the activity principle means that students are challenged with problem situations in which they gradually develop a formal way of problem solving based on informal ways of working, thereby constructing their own knowledge (Van den Heuvel-Panhuizen, 2000).

**Reality principle.** The RME approach focusses on enabling students to apply mathematics. In this sense, students are able to use their mathematical understanding as a tool to solve problems. In RME, the reality principle is not only recognisable in the area of application but also as a source for learning mathematics by using something *real* in relation to students’ thinking and imagination. Hence beginning a mathematics lesson with a rich and meaningful context is considered important for students to make sense and help them to develop mathematical tools and
understanding (Van den Heuvel-Panhuizen, 2000). In this way, students are in a better position to apply their understandings to solving problems in their real life situations.

**Level principle.** As pointed out by Van den Heuvel-Panhuizen (2000), learning mathematics involves students passing through different levels of understanding: from the ability to invent informal context-related solutions, to the creation of various levels of short cuts and schematisations. Models are suggested as an important device to help connect these different levels of knowledge from informal to formal mathematics. Hence, the use of appropriate models is considered to support students’ learning of mathematics and help them to reach various understanding levels.

**Inter-twinement principle.** Mathematics is not considered as a fragmented body of knowledge in which ideas exist isolated from the rest of knowledge. Instead, school mathematics is recognised as knowledge that should be integrated within and across other domains. This is to indicate from a deeper mathematical perspective that the elements within mathematics are connected both vertically and horizontally and cannot be separated (Van den Heuvel-Panhuizen, 2000). For instance, when solving rich context-based problems, students are expected to apply a broad range of mathematical tools and understanding, thereby bringing coherence to the curriculum. To cite an example, while solving a mathematical problem on fractions, students are expected to apply their knowledge and understandings related to ideas such as fair sharing, division, ratio, percentage, decimals and measurement.

**Interaction principle.** This principle focuses on the importance of thinking aloud during the learning situation. Students are encouraged to share their thoughts and learn from each other, thereby making the learning of mathematics a social activity. For instance, a teacher is expected to provide a thought-provoking task to students in groups so that they can share their thoughts and understanding with each other. In the process, interaction is said to evoke reflection which enables the students to reach a higher level of understanding (Van den Heuvel-Panhuizen, 2000).

**Guidance principle.** The teacher acts as a facilitator to guide the students towards understanding by creating an active learning environment in which students are encouraged to become independent of the teacher. For instance, while students are given a context based problem to solve on their own, a teacher is expected to be
well prepared and equipped with sufficient knowledge to steer the learning process by attending to the requirements of individual students. As described above, Hadi (2002) claimed that an RME-based approach to school mathematics provide some of the best and most comprehensive elaborations of the problem-based approach to mathematics education. This claim is based on Hadi’s work in introducing RME in Indonesia. In order to encourage and motivate the re-invention of mathematics in RME, students are provided with an appropriate context to work in. The selected context is taken either from the real world or from another area of mathematics with which students are familiar, and can make sense of.

The word realistic is used to highlight that students are able to imagine the situation in real and meaningful ways (Dickison et al., 2010). As argued by Johnson and Onwuegbuzie (2004, p. 7), “students see meaning in school work when they connect information with their own experiences”. Therefore, RME demands that teachers link the children’s own world and the world of mathematical ideas. In this way, the informal knowledge that children experience outside the classroom is recognised and acknowledged. Several studies have found RME to have a positive impact on students’ understanding of mathematics compared to the practices of more traditional approaches (Mullis et al., 2000).

After developing the RME-based mathematics curriculum, which strongly emphasised students making sense of the subject, the Netherlands became one of the higher mathematically achieving countries in the world (Hough & Gough, 2007). In addition, support of students’ achievement in mathematics through the RME approach was demonstrated in several international test results such as The Trends in International Mathematics and Science Study (Mullis et al., 2000). Unal and Ipek (2009) conducted a study regarding the effects of RME on seventh grade students’ achievements with respect to the multiplication of integers. The research was carried out with two different groups of seventh grade pupils in an Erzurum primary school. The results of this study indicated that “students who participated in RME learning activities were more successful performing integer multiplication compared with students participating in conventional teaching activities” (Ünal & Ipek, 2009, p. 2).

Further evidence was found by Dickison et al. (2010), who studied the use of RME with low to middle-attaining pupils in English secondary schools. They discovered that students taught using a RME-based curriculum were more able to
make sense of mathematics than by traditional methods, both in providing reasons for which they felt their answers were correct. According to Dickison et al. (2010), RME-based curricula, which focus on using imaginable learning task to help students actively develop mathematical knowledge, seem to have been more effective than traditional teaching approaches.

RME has also been introduced to reform the mathematics curriculum in developing countries. Sembiring et al. (2008, p. 2) have commented how the introduction of an RME-based curriculum in Indonesia (referred to as PMRI) has not only had an impact on student learning but also on the ways in which teachers teach:

> Mathematics education reform in Indonesia has been initiated in classrooms and teachers have changed their mathematics teaching approaches as a result of their involvement with new materials, textbooks, investigation, experiments, in-service education and in-class training.

PMRI’s bottom-up approach has supported both teachers and students in activity-based learning of mathematics using materials which have been largely developed in the classroom rather than behind the desks of curriculum officers. In this way, teachers feel a sense of ownership. They enjoy seeing students’ classroom experiences of materials and the tended to change their mathematical teaching approaches as a result (Sembiring et al., 2008). Although the RME approach is not directly referred to in the new Bhutanese mathematics curriculum, its application is consistent with NCTM standards and principles. As such, some of the key information regarding RME and its practical application was highlighted in this section as one of the two practical examples of the constructivist view of teaching and learning of mathematics. This was done mainly because of the new curriculum intentions, which are primarily based on NCTM standards and principles in terms of a practical approach in the mathematics classroom.

**Comparison of NCTM and RME approaches**

In terms of practicality and implementation of mathematical concepts in the classroom, the philosophy of RME tends to play an almost equal role in teaching and learning approach with NCTM standards and principles. A strong connection between RME and NCTM is clear, as pointed out by Torrence (2003, p. 90): “many aspects of RME are similar to the ideas of NCTM”. This similarity is highlighted in
Most of the characteristics of the social constructivist view of teaching and learning of mathematics are seen in both NCTM standards and RME principles. For instance, engaging students actively in constructing their own knowledge is considered to be the key element of the constructivist approach and is adopted in both the philosophy of NCTM (i.e., learning by doing) and RME (i.e., activity principle). This is to be achieved by creating situations for students to be involved on their own through the process of communication expressed in NCTM with RME’s interaction (Gravemeijer, 2004). Likewise, NCTM’s connects with RME’s concept of interwinement through horizontal and vertical connections. Students are encouraged to communicate their thinking through interaction with teachers and their fellow students, and by linking their knowledge of mathematics across topics and across other bodies of knowledge. The same philosophy is applied in NCTM standards.

A strong connection can be seen between the implications of RME principles and NCTM standards. For instance, with respect to learning, which is one of the NCTM principles, NCTM argued that learning is authentic when students are given opportunities to engage both physically and mentally in solving some mathematical tasks related to the targeted topic. In the process of doing the realistic activity (application of RME’s reality and activity principles), students are expected to think and share their ideas and justifications with fellow students and the teacher (RME’s interaction and intertwinement principle) and NCTM’s process standards (such as communication, connection and reasoning). According to NCTM (2000), teaching is expected to begin from what students already know. In this case, a teacher is expected to frame some authentic learning activity (RME’s reality principle) to
explore the existing knowledge of students based on the targeted mathematical topic (RME’s inter-twinement cum guidance principles), prior to the introduction of the new concept. While students are in the process of solving the given pre-concept task, a teacher can be expected to evaluate their level of knowledge in terms of connection and representation of ideas.

Similarly, NCTM’s assessment principle has its main focus on assessment for learning or formative assessment. According to NCTM (2000), teachers are expected to conduct assessment for learning parallel to their teaching to ensure students’ level of understanding before moving on to the next level. This process can be conducted through various assessment tools and models (RME’s level principle) rather than leaving it to the end (after completing a topic or a unit). Conducting assessment during the lesson is expected not only to assess students’ learning but also teachers’ teaching.

NCTM’s curriculum principle is defined to be the inclusion of authentic materials in terms of coherence between the class levels, use of simple language and citing realistic examples based on the readiness level of the students. In this case, it tends to relate appropriately to RME’s principles such as reality, level and inter-twinement which are applicable for the NCTM process standards (i.e., connection, representation and reasoning). Likewise, RME’s guidance principle can be related to the NCTM’s equity principle, where the teacher is expected to be mindful of catering to students equally and rendering support willingly as and when required through proper communication between teacher and students. Finally, NCTM’s technology principle influences the mathematics that is taught and enhances students’ learning. Using RME’s principles, such as activity, interaction, inter-twinement and guidance applied in collaboration with NCTM’s process standards of problem solving, communication and connection.

3.2.4 Challenges of NCTM standards and principles

Although there has been widespread advocacy of the NCTM standards there have been criticisms of the standards and principles on which they are based. According to Bosse (1995), the NCTM standards approach is “considered successful when it is seen as a visionary tool and less success is ascribed to them when its implementation is considered” (p. 181). The philosophy of NCTM is consistent with the constructivist view of learning mathematics, where learners are expected to construct
their own knowledge based on their previous knowledge and experiences (Lefrancois, 1997).

However, Kittel (1957) argued that when comparing the performance of students using the pure discovery approach to guided discovery, the pure discovery group performed poorly. Indeed, cognitive load theorists argue that constructivist approaches based on a discovery learning are ineffective compared with instruction that provides direct, explicit information (Sweller, Kirschner, & Clark, 2007). According to these authors, there is overwhelming evidence supporting the strategy of providing a learner with a problem solution. A direct instructional approach enhances learning compared with strategies based on students having to discover the solutions themselves (with or without assistance through scaffolding). There is another group who emphasise scaffolding, in the sense of providing learners with appropriate prompts and hints needed to complete problem solution (Hmelo-Silver, Duncan, Chinn, & 107., 2007; Schmidt, Loyens, van Gog, & Paas, 2007).

A compromise advocated by Sweller, Kirschner and Clark (2007) requires teachers to precisely determine the amount and type of guidance required by different learners through the careful design of instructions. In this case, learners can still have the opportunity to construct their new knowledge but with careful and guidance from the teacher.

As argued by Mayer (2004), when students have too much independence, they may fail to actively engage with the material to be learnt. Rather, Mayer (2003) considers learning to be meaningful when learners endeavour to make sense of the presented material by selecting appropriate incoming information, organising it into coherent structure, and integrating it with other organised knowledge. Thus, Mayer (2004) argued that “the constructivist view of learning may be best supported by methods of instructions that involve cognitive activity rather than behavioural activity, instructional guidance rather than pure discovery, and curricular focus rather than unstructured exploration” (p. 14).

As discussed above, some of the failures of NCTM Standards must have encouraged the USA in the past five years to move away from NCTM’s (2000) standards. Further, one reason may be poor US performance in the middle grades on the Trends in International Mathematics and Science Study (TIMSS) international
assessments. USA students ranked 16th out of 46 participating nations, during the TIMSS-2003, whereas Singaporean students stood first in the same test. When compared with the Singapore mathematics framework, although the NCTM framework emphasizes high order, 21st century skills in a visionary way, it lacks the logical mathematical structure of the Singapore model (American Institute for Research, 2005).

3.2.5 Summary

In this section, a review of literature on constructivist theories and practices of mathematics education has been presented. Two examples based on constructivist theory were presented: NCTM principles and process standards; and the RME principles. The Bhutanese mathematics curriculum is based on NCTM standards. RME is included because it is consistent with NCTM standards practised in the classroom. In fact, RME principles can provide concrete links to practice. Aligning ideas from NCTM and RME, a brief account of the implications of the reformed curriculum and its challenges is presented in the following section.

3.3 CHALLENGES OF CURRICULUM REFORMS

The ultimate goal of a reformed mathematics curriculum is to improve students’ learning through change of the practice advocated in the reform. According to Chan and Luk (2013), curriculum reform includes changes in the educational system, program structures and intentions, which lead to changes in approaches to teaching and learning, and students’ learning results. Students, facing the challenges of the global market, are required to use their education qualifications as vehicles for employment and further studies, often outside their home countries. For this reason, an urgent need was felt for a standardised way of recognising qualifications through reforms (Chan & Luk, 2013). Charalambous and Philippou (2010) argued that teachers are considered key role players in executing curriculum reform and they emphasised providing teachers with sustained support lest they fall back to a comfort zone of existing routines and practices. Furthermore, according to Keys (2005), teachers’ beliefs are a major influence on the implementation of any curriculum initiative. Nevertheless, research has been conducted on how teachers’ beliefs influence the implementation of the curriculum. In several studies conducted on the implementation of reformed curricula, the teachers’ journey appears to be not
smooth, particularly with mathematics curriculum. For instance, verbally, teachers may agree with curriculum change but rarely do their words match their classroom practices (Keys, 2005).

Consequently, there commonly exists a mis-match between intended, executed and accomplished curriculum (Cuban, 1993; Handal & Herrington, 2003; Howson & Wilson, 2012). Although, the intended curriculum is set by policy makers, the execution of curriculum is carried out by teachers in their classrooms. According to Chissick (2005), a few pertinent issues can be identified which can be further grouped into two conditions, namely field conditions (e.g., time, administration’s support, teamwork and peer support, external intervention, external examinations) and psychological conditions (e.g., teacher’s self-esteem, belief system and recognition of the needs for change). Chissick (2005) highlighted the importance of a resilient, cohesive and supportive school team under good leadership as a vital starting point for the effective implementation of reforms.

Further, one of the most common issues affecting the implementation of a reformed curriculum is the existing beliefs and practices of teachers. Teachers are a critical component to reform as they decide whether or not to implement the reform in their classrooms (Beck, Czerniak, & Lumpe, 2000). Roehrig and Kruse (2005) argued that “it is evident that teaching beliefs played a large role in the nature of implementation of reformed based curriculum” (p. 420). For instance, in their study, teachers holding primarily traditional beliefs displayed little change in their classroom practices and a low level of implementation of the curriculum. Beside teachers’ beliefs, Roehrig and Kruse (2005) identified teachers’ content knowledge as affecting implementation but, this was seen as secondary to beliefs. Thus, teachers are key to the success of curriculum reform (Guskey, 1995; Smith & Desimone, 2003; Spillane & Callahan, 2000).

As argued by Riordan and Noyce (2001), curriculum materials alone cannot change teachers’ practice without the provision of timely and ongoing scaffolding in trying to create a classroom environment, aligned with curriculum intentions. Provision of support to teachers was found to help develop teachers’ knowledge and beliefs, which play a crucial role in understanding the reforms (Blignaut, 2007; Haney, Lumpe, Czerniak, & Egan, 2002). Having failed to provide timely support to teachers, it would be illogical to expect teachers to accept educational reforms such
as a reformed mathematics curriculum without resistance (Bantwini, 2010; Mohammed & Harlech-Jones, 2008).

Furthermore, the implementation of a standards-based curriculum has been found to depend on teachers’ beliefs and practices associated with a constructivist approach of teaching and learning of mathematics (Keys, 2007). As shown in the examples discussed in Section 3.2.3, teachers with a deep understanding of curriculum reform associated with the constructivist approaches were found to positively influence the learning of their mathematics students. For instance, Ma (2005) conducted a survey in reform classrooms and found that students in those classrooms were encouraged to communicate their views, illuminate their ideas, and respond to the ideas of their classmates. However, according to Li and Ni (2011), one of the challenges faced by teachers was engaging students constructively. Most of the time, teachers were inclined to pressure students to agree with them or permitted students to do whatever they liked without proper guidance, feedback or instructions. At the same time, group work was found to be ineffective in terms of purpose, time limits or guidance from the teachers (Yu, 2003).

Moreover, in a study conducted on the impact of curriculum reform by Li and Ni (2011), compared to procedure with connections, without connections was found to be more popular in terms of recurring tasks. Likewise, questions asking for an explanation were more often asked by reform oriented teachers than non-reform teachers. In addition, Cai (2005) argued that exposure of students to meaningful and cognitively demanding tasks has the potential to provide authentic prospects to explore mathematical ideas and as a result, develop aptitude in mathematics. For this, a teacher must have a strong entrenched belief about mathematics and mathematics education associated with a constructivist view of teaching and learning of mathematics. Nevertheless, as argued by Hsu and Wang (2012), just knowing the proposition of a theory may not change teaching behaviours.

Implementation of a reformed curriculum is often found to be ineffective when teachers hold beliefs that are in disagreement with the objectives of the curriculum. This indicates a strong requirement for conversion of teachers’ beliefs in terms of understanding teaching and the learning of subject matter aligned with reformed curriculum materials (Powell & Anderson, 2002). Failure to transform teachers’ beliefs creates a gap between the enacted and intended curriculum (Goodrum,
Hackling, & Rennie, 2001). This could be the consequence of teachers’ paying more attention to covering the syllabus using a transmissive approach at the expense of student learning through more inventive and reformed-based practices (Roehrig & Kruse, 2005). As concluded by Roehrig and Kruse, teachers need time to understand the reformed based curriculum and therefore required rigorous one-on-one professional learning activities. They found that, over an extended time period some teachers challenged their beliefs and embraced the new curriculum. Thus, a major reason for the limitation of an effective implementation of curriculum reform was found to be the influence of teachers’ beliefs (Driel, Beijaard, & Verloop, 2001; Keys, 2007).

As discussed earlier in this chapter, Keys suggested a three types of teachers’ beliefs labelled as expressed, entrenched and manifested. Klein (1997) argued, that the exhibition of teachers’ epistemological beliefs is articulated through their practised knowledge. There is a growing acknowledgement of the role of the teacher in curriculum change and as a result the literature indicates a need to better understand teachers’ epistemological beliefs in determining curriculum effectiveness (Driel et al., 2001). Therefore, as argued by Keys (2005), understanding teachers’ beliefs and connecting these with initiators of change could limit the gap between the intended and the enacted curriculum.

According to Fullan (2001), barriers to the implementation of standards based curriculum were associated with traditional, teacher-centred beliefs, teachers’ knowledge bases, and personal characteristics (i.e., age, teaching experience, academic training), and the traditional organisational structure of schools. Roehrig and Kruse (2005) reported that, although the curriculum intended for the class time to be divided equally between whole class, small group, and individual work, teachers were observed spending 70% of time giving direct instruction to the whole class. The justification given by the participating teachers was “students only learned with the help from teachers and through repetition” (Roehrig & Kruse, 2005, p. 413). Several studies have revealed that in order to bring about successful reform within the classroom, the concerned authorities (policy makers) are required to have an understanding of the course of change within the classroom teacher (Hopkins, 2001; Keys, 2003, 2005).
Keys (2007) discussed the absence of a model to depict a change taking place at the individual level when curriculum is enacted. Keys also made clear the importance of understanding of how a teacher implements the curriculum. Keys (2007) suggested a model through which to expedite the course of change within the individual teacher, and thereby support effective curriculum reform. At the same time, several studies have revealed that teachers struggle to implement a reformed curriculum effectively.

In a review of the public health literature (Dane & Schneider, 1998; Dusenbury, Brannigan, Mathea Falco, & Hansen, 2003), five criteria for measuring fidelity of implementation were identified: adherence, duration, quality of delivery, participant responsiveness, and program differentiation. These criteria can be applied in measuring the implementation of any reform including reformed curricula, for instance, to evaluate whether the components of the reformed curriculum are being delivered as designed (adherence), the manner in which the implementer delivers the program using the techniques, processes, or methods prescribed (quality of delivery), and the extent to which participants are engaged and involved in the activities suggested in the reformed curriculum (participant responsiveness). In addition, sufficient time for implementing the reform was another issue that emerged in terms of effective implementation of the reforms, along with the presentation. According to Hall, George, and Rutherford (1977), teachers move through seven stages of concern as they adopt reforms: awareness, informational, personal, management, consequences, collaboration, and refocussing. The first stage is suggested as awareness, where teachers feel they know about the reform but lack interest in learning more about it. The informational stage is reached when they start taking interest in the reform and began to implement it based on their capabilities (personal stage), and when considering the existing organisation and logistics of reform (management stage). Gradually, when a teacher starts observing the impact of reform on students’ learning (consequences), they tend to share their ideas and findings with their colleagues (collaboration). Finally, the focussing stage takes place when teachers take the initiative to modify and improve further on the reforms.

As presented above, each of the stage appeared to be useful in terms of teacher development aligning with the intentions of curriculum reforms (Hall et al., 1977). Of all the stages, moving a teacher from awareness to informational will be very
challenging if favourable conditions and resources are not provided. The next element is reaching the stage of consequences and moving to refocussing, where teachers can become inspiring and productive in terms of contributing to the education system as a whole. Moreover, as concluded by McKinney, Sexton, and Meyerson (1999), from their study on validating the efficacy-based change model, the success of a reform depends on teachers’ moving from self-concerns to task concerns (Van den Berg, Sleegers, Geijsel, & Vandenberghe, 2001).

As proposed by Fuller (1969), three levels of teachers’ concerns are self, task and impact. Self-concern is about teachers’ anxiety in terms of their ability to successfully undertake the new demands stemming from the reforms. Task concerns are associated with teachers’ daily duties in terms of time constrictions to cover the syllabus in time, limited resources and having to deal with a large number of students in the class. Finally, impact concerns are about the significance of the change to student learning. However, as argued by Tunks and Weller (2009), a successful shift from one concern to another is facilitated only when teachers are unceasingly and substantially reinforced in implementing the reform. Often, teachers implementing reforms complained that the time allotted was insufficient, and as a result, scaffolding students in time is almost impossible (Charalambous & Philippou, 2010).

VanBalkom and Sherman (2010), from the Faculty of Education, University of Calgary, Canada, conducted a study regarding teacher education in Bhutan, which highlighted challenges for reform, focussing on the two teacher training colleges. Some of the major themes that emerged from their study were issues on the quality of students and teaching, the integration of theory and practice, organisational structure and culture, and working conditions at the colleges, and in the broader education sector. In the process, the prevailing concerns regarding the implementation of the new curriculum in Bhutan were included in the list of themes that emerged. According to VanBalkom and Sherman (2010), the prevailing concern regarding the implementation of the new curriculum in Bhutan is unchanged approaches in teaching at colleges and schools, which defeat the purpose of bringing lasting and fundamental changes to the teaching and learning of mathematics. VanBalkom and Sherman (2010), concluded that one of the main reasons for this seems to be a lack of experts with a sound background in terms of subject knowledge and rich teaching experience, who can actually demonstrate the use of the reformed
ideas inside the classroom. The module tutors in the Colleges of Education at best taught the theory in part only, and actual practical experience seemed to be weak during the teacher-training period.

The prevailing problem is with the way teachers implement the intentions of the new curriculum in the actual mathematics classrooms. With a limited opportunity for professional learning activities related to the use of the new mathematics curriculum, it is questionable whether the ideas intended in the curriculum are actually being implemented. Consequently, there is a need for this study to create more awareness among the relevant authorities. In response to this, timely professional development support may be given to teachers to generate maximum benefits for students. Aligning with some of the pertinent issues discussed in this chapter to help address the four Research Questions, a theoretical framework of this study, formed by three main issues (i.e., beliefs, social constructivist theory and challenges of curriculum reforms) was reviewed, which was further extended to develop a conceptual framework, which is presented in the following section.

3.4 CONCEPTUAL FRAMEWORK

As discussed earlier in this chapter, the theoretical framework for this study is based on the importance of teachers’ beliefs in the conduct of the mathematics lessons in the classroom. For instance, a teacher with a belief aligned to constructivist approaches to teaching and learning of mathematics would be expected to conduct lessons featuring characteristics associated with that belief. From the theoretical framework, a conceptual framework was developed as presented in Figure 3.4.
Figure 3.4, identifies the relationship between teacher beliefs about the subject (content) and teacher beliefs about learning. If a teacher has beliefs leading to an experimentalist philosophy and that learning occurs through social interactions, then the curriculum adopts the principles of social constructivism. These include the learner as active inquirer learning in context and culture through language and dialogue. The classroom is learner-centred. If the teacher or system adopts instrumental beliefs in learning (not shown in the diagram), then the learning environment would be focussed on delivery of content, with students assuming a passive role learning rules and procedures. The learning environment would be teacher-centred, creating a mis-alignment with the curriculum intentions, as presented in Figure 3.5.
As illustrated in Figure 3.5, beliefs about mathematics could be measured in terms of teachers’ and students’ practice aligning or mis-aligning with the curriculum intentions. The column on the left presents different types of beliefs and the column on the right indicates the extent of alignment or mis-alignment of the curriculum intentions. For instance, if both teachers and students practice support experimentalist beliefs about mathematics, then, it is considered to be aligned with the curriculum intentions. In contrast, if both the teachers and students practice reflect instrumentalist beliefs, then, such practice is considered to be mis-aligning with the curriculum intentions. However, in the case of Platonist, teachers’ practice then there is mis-alignment with the intentions of the curriculum.

Bhutan has adopted a constructivist curriculum based on the NCTM standards. One can summarise these principles in terms of the content and processes of the curriculum. The content is captured through an emphasis on relational understanding, reasoning and its context embedded in the form of the first three curriculum intentions. The fourth intention can be represented as a window into the classroom through the nature of content or tasks that the students engage in: solving problems and applying the other process standards (connection, communication, representation, reasoning and proof). Because in the conceptual framework, learning is seen as situated and occurring through action and social dialogue, the research adopted the methodology outlined in Chapter 4 of survey, design of sample learning activities and observation of classroom practices to examine teachers’ beliefs about mathematics and any contradiction between beliefs and practices. The detail of these processes is not evident in the diagram but are captured by the content of the upper box in the conceptual framework. The combination of teacher’s beliefs about the content and students’ learning contributed in framing the 3+1 analytical framework.

Figure 3.5. Teacher’s beliefs about mathematics and curriculum intentions.
out of the four intentions of the new curriculum. This analytical framework is intended to compare teacher intentions and practice and a brief account of its development is presented in the following section.

3.5 **ANALYTICAL FRAMEWORK**

This section draws on the literature review to propose an analytical framework of this study. Although the new Bhutanese curriculum is based upon the NCTM principles and standards, it was found reasonable to draw upon the RME principles to enrich the descriptions of the curriculum’s four intentions. This section draws together ideas from the NCTM and RME to form an analytical framework based upon the study’s conceptual framework. The analytical framework is termed the *3+1 framework*, which is composed of the first three intentions (emphasis on understanding, reasoning and use of context) as ‘3’ and ‘+1’, the fourth intention (use of process standards). The fourth intention is termed as ‘+1’ as this intention relates to the extent to which student activity is be woven into the achievement of the first three intentions, as presented in Figure 3.6.

![Figure 3.6. Concept of 3+1 framework.](image)

This 3+1 framework provided the base upon which to study the practices of Bhutanese teachers, and to explain these practices in terms of the teachers’ beliefs regarding mathematics and mathematics education. In the following sub-sections, each of the first three intentions is discussed in relation to the fourth, and an analytical rubric for each is proposed. Each takes the form of the general rubric comprised of two columns and two rows, as shown in Table 3.3.
Table 3.3  
*Structure of the general rubric for analysis*

<table>
<thead>
<tr>
<th>Indicators</th>
<th>What Teachers Does</th>
<th>What Students Do</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Descriptions of aligned teachers’ practices with regard to curriculum intentions</td>
<td>Descriptions of students’ engagement aligned with curriculum intentions</td>
</tr>
<tr>
<td>Counter-indicators</td>
<td>Descriptions of teachers’ practices mis-aligned with curriculum intentions</td>
<td>Descriptions of students’ engagement not aligned with curriculum intentions</td>
</tr>
</tbody>
</table>

The column on the left is presented with *indicators* and *counter indicators* to represent the performance of the teacher (what the teacher does) and students (what students do) during the lesson. For instance, if the performance of a teacher is aligned with the targeted curriculum intention, it is presented under *indicators* and if otherwise under *counter indicators*. The same format was used for the respective descriptions for each of the first three curriculum intentions. The fourth intention is expected to have been integrated in the processes by which students perform learning tasks.

3.5.1 **Intention One: Emphasis on understanding of mathematical concept**

The first curriculum intention is about teaching mathematics for deep (relational) understanding. According to Skemp (1976), relational understanding is “knowing both what to do and why, rather than ‘what’ alone” (p. 2). It is considered as a kind of mental schema in which knowledge is constructed through the network of connections within and across the world of knowledge: the more the connections, the better the understanding. This idea of connection is evident in NCTM connection standards and RME’s two ways of mathematisation (i.e., horizontally and vertically). This intention focuses on fostering students’ conceptual understanding of mathematics by providing opportunities to identify and develop connections with their previous knowledge (RME’s vertical connection) and experience outside the class (RME’s horizontal connection).

Aligning with this idea of vertical and horizontal connections, Gadanidis and Hughes (2011) present the idea of mathematical connections through their investigation of big ideas in mathematics. Over a period of six months, they worked in partnership with a group of teachers of Grades K-8. They discovered that *Big math*
ideas can be addressed across grades, at different levels of mathematical sophistication in terms of vertical connection and through horizontal connection. This is especially the case when learning activities are designed to have a low mathematical floor (allowing students to engage with minimal mathematical knowledge) by relating to students’ immediate life-based activities, at the same time providing a life-based but thought-provoking experience with a high mathematical ceiling for students to extend their mathematical thinking. In this way, students are exposed to the construction of knowledge leading towards relational understanding, specifically the cognitive part of internal representation. As stated by Barmby, Harries, Higgins, and Suggate (2007), “in making links between understandings of a mathematical concept through reasoning, for example showing why 12 x 9 gives the same answer as 9 x 12, we further develop our understanding of the concept” (p. 42). Doing this, students are able to assign meaning and purpose to mathematical processes and properties and institute connections among mathematical topics (Fillingim & Barlow, 2010).

Further, Alba (2001) argued that “mathematics must be meaningful if students are to communicate and apply mathematics productively” (p. 70). In order to implement this, there are certain expectations for both teachers and students (National Council of Teachers of Mathematics, 2000; Treffers, 1987). Every action performed by a classroom teacher tends to have some impact on what students do. For instance, if a teacher tries to present a thought provoking activity, students are expected to become actively engaged to construct their own knowledge. As argued by An, Ma, and Capraro (2011), connecting existing knowledge to new knowledge provides opportunities for students to make sense of mathematics and apply mathematical knowledge in meaningful ways. In order to bring this idea into reality, a teacher is recommended to undertake specific classroom practices such as the use of cooperative learning approaches (interaction principle), enquiry based activities and the use of materials and manipulatives to nurture a meaningful illustration of mathematical concepts (McCaffrey et al., 2001).

A teacher having adopted the use of cooperative learning approaches should have implemented the intention of the new curriculum, which is strongly rooted in NCTM standards, particularly with connection and communication. These ideas are apparent in RME principles such as the interactive principle, which encourages
students to communicate and connect their thinking and understanding with others to see mathematics more globally. This point is strongly supported by Matthews (2000), who argued that “students should emerge from school mathematics with a global and interconnected view of mathematics, not a compartmentalized view” (p.39). This emphasis recognises the pervasive notion that mathematics should be experienced by students as a network of inter-related concepts and procedures, rather than a collection of isolated rules and facts (Brinkmann, 2003). Hence, a lesson should nurture students’ conceptual understanding of mathematics as well as building their problem solving skills (Perrin, 2008).

According to the National Council of Teachers of Mathematics (2000) standards, to foster in students a deeper understanding of mathematics, a teacher is expected to organise lessons in such a way that students see how mathematical ideas build on and connect with one another. This expectation also relates to the RME’s level principles. For example, one strategy encouraged for such lessons is to design them around one central mathematical big idea, which is carefully developed and extended (Gadanidis & Hughes, 2011). To this end, students are encouraged to be engaged with a context-based problem, to explore and construct their own knowledge through the application of the process standards (National Council of Teachers of Mathematics, 2000). This is especially the case when learning activities are designed to have a low mathematical floor and high mathematical ceiling. These characteristics of problems enable all students to engage with the relevant mathematics and therefore develop their understandings. This idea of students’ own production of knowledge plays an important role in RME (Van den Heuvel-Panhuizen, 2000).

The following indicators can be used as evidence of students’ relational understanding while performing given tasks:

- Verbal indicators (students being able to explain and share their thinking and understanding with either teacher or fellow students);

- Cognitive indicators (students being able to connect and apply previous knowledge and experience to solve the current problem or task); and

- Physical indicators (students being able to represent their understanding in various forms (i.e., enactively, iconically and symbolically).
Having provided the opportunities for students to depict their understanding as indicated above, the consequences expected are leading students to relational understanding, thereby helping them to gain deep conceptual knowledge. However, the success of implementation could depend on the type of beliefs held by the teacher. For instance, a teacher holding beliefs about mathematics as dynamic and connected within and across the curriculum could provide students with an opportunity to explore and construct mathematical knowledge themselves (i.e., beliefs about learning mathematics).

In the process, students create an avenue for the teacher to identify the next stages of learning to be filled rather than the teacher ‘spoon feeding’ them without recognising their needs. This strategy has been integrated in the new Bhutanese curriculum in the form of introducing the *Try This* activity at the beginning of the mathematical lesson. Having performed this activity, the depth or degree of students’ understanding of the targeted concept is determined, as stated by Hiebert and Carpenter (1992, p. 67), “by the number of and strength of its connections”. At the same time, students are expected to be able to represent their understandings through various modes: enactively, iconically and symbolically (Bruner, 1966).

In Table 3.4, the rubric is proposed for analysing teaching and learning in regards to the first and fourth intentions of the new curriculum. Of the three columns, the middle one describes teacher’s action and to the right, students’ engagement during the lesson. The top row describes indicators and the bottom row counter indicators in terms of actions performed by teacher and students. The top row describes teacher and student actions which align with the intentions (i.e. indicators), while the bottom row describes teacher and students’ actions which do not align with the intentions (i.e. counter-indicators). As discussed earlier, the fourth intention is expected to be integrated mainly in the process of actions performed by students. The fourth intention (use of process standards) is attained when students implement tasks.
### Table 3.4

**3+1 rubric for relational understanding**

<table>
<thead>
<tr>
<th>Indicators</th>
<th>What Teachers Does</th>
<th>What Students Do</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Use learning activities that explore and scaffold construction of conceptual schema:</td>
<td>Participate in activities in which they identify and describe the developing horizontal and vertical connections within their own conceptual schema</td>
</tr>
<tr>
<td></td>
<td>- Horizontally between students’ reality (i.e., daily-life activities or situations that can be readily imagined by the students) and mathematical activities</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Vertically between mathematical ideas of varying levels of abstraction or sophistication</td>
<td></td>
</tr>
<tr>
<td>Counter-indicators</td>
<td>Use of learning activities, which do not incorporate opportunities to explicitly explore the connections between mathematical ideas</td>
<td>Passively listen to teacher explanations</td>
</tr>
<tr>
<td></td>
<td>Do not describe the vertical and horizontal connections they perceive between mathematical ideas</td>
<td></td>
</tr>
</tbody>
</table>

#### 3.5.2 Intention Two: Emphasis on reasoning

The second intention of the new curriculum emphasises reasoning. This term refers to students being able to justify their thinking and understanding of the taught mathematical concepts. Students are expected to reason out their understanding through various forms of representations such as enactively (in action), iconically (in visual) and symbolically (symbols). In support of this, Bruner (1966) argued that students need to be given opportunities to communicate their thinking and understanding in various forms of representation (enactive, iconic & symbolic) and justify (reason) their derived solution in many possible ways. Bruner’s ideas of representation are strongly integrated as one of the NCTM process standards (i.e., representation), from which students’ learning can be graded into different levels of knowledge. This idea is closely aligned with RME’s level principle, which underlines that learning mathematics means students achieving increasing levels of cognitive understanding (Treffers, 1987). Similarly, Matthews (2000) argued that learning to reason is considered essential to doing mathematics. Moreover, National Council of Teachers of Mathematics (2000, p. 20) states that “learning is often fragile when students attempt to learn by memorizing facts or procedures without understanding”. Rather, the NCTM (2000) suggests that students should learn to make conjectures, gather evidence and build arguments to support their ideas. According to Miura (2001), this type of understanding is referred to as internal,
derived from the external in the form of symbols, words and pictures. The process of internal understanding is said to be mediated through external representations such as spoken language, written symbols, pictures and physical objects. Alba (2001) argued that instructional practices aligned with these process standards are more effective in developing students’ mathematical reasoning than traditional instructional practices.

The emphasis on reasoning was adopted in the new curriculum to encourage students to explain their thinking and mathematical ideas. For instance, when students are given the opportunity to express their thinking and justify their understanding through various modes of representation (enactive, iconic and symbolic), this is considered to enhance the learning situation for both teacher and students. For example, the evidence of why $\sqrt{2}$ is an irrational number can be achieved through the manipulation of symbols and rational statements (Barmby et al., 2007). Table 3.5 presents the analytical rubric for analysing teacher and student actions in regards to the second and fourth intentions of the curriculum.

<table>
<thead>
<tr>
<th>Indicators</th>
<th>What the teacher does</th>
<th>What students do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use thought provoking learning activities which encourage students to explain their thinking and mathematical ideas.</td>
<td>Explain their thinking and mathematical activity.</td>
<td>Flexibly use of written and spoken language as well as symbolic, iconic and enactive representations.</td>
</tr>
<tr>
<td>Integrate a range of written and spoken language and representations (enactive, iconic and symbolic) to express mathematical ideas.</td>
<td>Focus on achieving correct answers without explaining or justifying the processes used.</td>
<td>Use rules and formulae without explaining why and how these rules and formulae work.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Counter indicators</th>
<th>Emphasis correct answers over the processes by which the answers were created.</th>
<th>Do not flexibly use a range of language and symbolic, iconic or enactive representations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Encourage reliance on ready-made rules and formulae without helping students to understand the origin of these rules or formulae.</td>
<td>Use rules and formulae without explaining why and how these rules and formulae work.</td>
<td></td>
</tr>
<tr>
<td>Primarily uses symbolic representations to express mathematical ideas.</td>
<td>Do not flexibly use a range of language and symbolic, iconic or enactive representations.</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3.5

**3+1 rubric for reasoning**

3.5.3 **Intention Three: Use of meaningful contexts**

The third curriculum intention strongly emphasises the provision of learning activities relevant to students. According to the National Council of Teachers of Mathematics (2000), students should learn mathematics in classrooms in which the emphasis is on thoughtful engagement and meaningful learning. Whitelegg and Parry
(1999) argued that context based learning is necessary in order to encourage students for high quality learning. Moreover, Maslow (1970) argued that the use of context based teaching has potential to help the learning process become less didactic; more negotiated; and promotes students’ self-esteem in learning the subject. For instance, a successful story is shared by Lowrie and Logan (2006), in their study conducted on using spatial skills to interpret maps which provided students with the opportunity to engage in realistic activities and ensure that mathematical concepts can be taught and expressed in contexts closer to students’ own experiences. Hence, as argued by Lesh and Harel (2003), the kind of problem solving situations provided in the classroom should be closely connected to the real-life experience of students.

This intention strongly aligns not only with NCTM standards but also with the philosophy of RME, which is based on Freudenthal’s (1977) view of mathematics as being connected to reality and relevant to society by enhancing students’ human values. The NCTM (2000) standards stress the importance of engaging students in solving real-world contexts and problems. According to Hill et al. (2008), real-world contexts for mathematics are thought to contribute significance and meaning to mathematics, so it is less abstract and more connected to the world. The importance of realistic problems is viewed as relevant and appealing to students as it relates to their daily experiences (Lesh & Harel, 2003; Perrin, 2008). The nature of such tasks is that they are intended to provide learners with a way to communicate their thinking processes by sharing and interacting with peers and teachers. As argued by Barnes (2005) “it is a common understanding that most people are less resistant to learning something new when they can see the purpose or meaning of it” (p. 46). De Corte (1995) pointed out that “learning essentially occurs in interaction with social and cultural contexts” (p. 41). Such an approach requires teachers to develop problems around contexts that are familiar, realistic and meaningful to students (Reusser, 2000).

Teachers are encouraged to design learning activities based on students’ previous knowledge and experiences, an approach which is directly linked with the NCTM’s curriculum principles and RME’s reality principles. Further, RME expects a teacher to introduce mathematics in context by choosing a problem that is realistic to a child, who can then understand and be motivated to solve it and in the process, develop mathematics (Torrence, 2003). According to RME, ‘realistic’ is not simply
its connection with the real world, but is related to problem situations which students can imagine (Heuvel-Panhuizen, 2000). One strategy involves connecting the given task vertically to previously learnt mathematical concepts, and horizontally with the children’s experiences outside the classroom. Thus students are dynamically involved in solving problems and building their own meaning and understanding (Barnes, 2005). According to Tarmizi et al. (2010), humanising mathematics is necessary to help students enjoy mathematics, transfer their problem solving skills to new situations and enhance their mathematics performance. This point is consistent with the NCTM (2000), which stated that students should have steady experiences with problems that connect to the real world and which interest, challenge and engross them in thinking about mathematics. Likewise, Gadanidis and Hughes (2011) argued for the importance of providing opportunities to see connections both within and beyond mathematical knowledge. This particular idea of using meaningful contexts was adopted by the new Bhutanese curriculum in the form of the Try This activity at the beginning of every lesson, where mathematical ideas are embedded in meaningful contexts.

As described earlier, in order to address this idea, the new curriculum has suggested some context related tasks (word problems) in several of the lesson components: The Try This activity in the beginning of a lesson and practice and applying at the end of the lesson. The nature of the Try This activity intends to be meaningful and realistic for the students, providing opportunities to apply process standards intended in the new curriculum. For instance, the example of a Try This activity related to fractions as division presented in Section 2.2.3 is intended to encourage the students to connect with their previous knowledge of whole numbers. Having stated that, it is very important for teachers to possess the kind of belief about mathematics teaching which matches such statements, and to be committed enough to provide similar types of problems. Thus, students become motivated to solve such problems with interest, are able to use all possible process standards, and are expected to improve their relational understanding and gain more conceptual knowledge.

As discussed in Chapter 2, the learning of mathematics in Bhutanese classrooms has been limited in terms of the emphasis placed on contextualised learning, mainly due to the adoption of a curriculum based on Indian culture. For
instance, examples cited for teaching mathematics such as speed, time, distance were all related to the train and its function, although trains did not exist in the life of Bhutanese students. So, the only alternative to solving such problems was by memorising the rules and formulas blindly without understanding them. In contrast to the above experience, the reformed Bhutanese mathematics curriculum emphasises the importance of contextualising learning. Contextualised learning assumes that teachers design rich learning experiences to ensure that learning activities relate to students’ real lives: one of the methods intended is to connect the given task vertically to their previously learnt mathematical concepts, and horizontally with the children’s experiences outside the classroom. Hence, as argued by Bickmore-Brand (1998b), the focus in the mathematics classroom is expected to be the crossing between how students recognise mathematical ideas in actual world contexts and the mathematical world presented in the classroom. Table 3.6 presents the analytical rubric for analysing teacher and student actions in regard to the third and fourth intentions of the new curriculum.

Table 3.6

<table>
<thead>
<tr>
<th>3+1 Framework on Use of Context</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What the Teacher Does</strong></td>
</tr>
<tr>
<td>Indicators</td>
</tr>
<tr>
<td>Counter - indicators</td>
</tr>
</tbody>
</table>

Tables 3.4, 3.5 and 3.6 have described the role that the teacher plays and the corresponding activities of students. For example, evidence of how teachers design learning activities in terms of helping students in deeper and meaningful understanding of mathematics is provided, while the corresponding behaviours of students are depicted. Within the discussion of each, the desired actions of the teacher and of the students are described. Thus, in the case of the students, this discussion is expected to embed the relevant aspects of the fourth intention of the curriculum. The types of roles played by teacher and students are measured in terms of alignment and mis-alignment with the philosophical intentions of new curriculum.
3.5.4 Research Aims and Questions

As discussed in Chapter 2, Section 2.3.1, the aims of the new curriculum are in line with the Bhutanese Ministry of Education’s policy document *Purpose of School Education for Bhutanese Schools PP-8* (Ministry of Education, 2002). In that document, the Ministry of Education has clearly stated that students should be provided with learning experiences that include frequent opportunities to explore, discover and describe mathematical patterns and relationships, and that this should be achieved by involving students in problem solving. The idea of involving students in problem solving connects closely with designing learning activities having a low mathematical floor and a high mathematical ceiling (Gadanidis & Hughes, 2011).

However, in the past, mathematics in the school was learnt through a traditional approach, aligning with the practice followed by a teacher with instrumental beliefs, as discussed in Section 3.1.2. The same practice was followed by students studying in the monasteries, where they learnt high-level mathematics in the name of *tsig* (astrology), mainly through memorisation of text and rules.

As with the introduction of any new curriculum, especially one that requires a significant change to teaching and learning practices, there could be a difference between the intended and enacted curriculum, that is, a gap between what the authors of the new curriculum designed, compared to what the classroom teacher delivers. Moreover, since the introduction of the new Bhutanese curriculum, no study has been attempted to evaluate its effectiveness compared with teachers’ beliefs and practices.

Recognition of the importance of teachers’ beliefs on mathematics education reform has underpinned significant research over recent decades (Beswick, 2007). Mathematics teaching reform normally requires deep changes in the system of beliefs held by teachers (Ernest, 1989). Teaching reforms can only take place when teachers' deeply held beliefs about mathematics teaching and learning are changed. It is therefore unreasonable to attempt to change the practice of teachers without changing their beliefs (Beswick, 2006b). Hence, this study aimed to explore this phenomenon in detail by looking into the practices of Bhutanese mathematics teachers and comparing those with the intentions of the curriculum.
To address the study’s aim, four Research Questions were identified (and were previously introduced in Chapter 1):

Question 1: What are the beliefs of Bhutanese primary teachers about mathematics teaching?

Question 2: What are Bhutanese primary teachers’ planning and classroom practices in teaching mathematics?

Question 3: To what extent are mathematics teaching practices aligned with the curriculum intentions?

Question 4: What influences the implementation of the intentions?

The first Question called for research which explores the types of beliefs that primary-school teachers hold about mathematics and mathematics education. The second Question asked for research into teachers’ actual practices in the classroom and impact of teachers’ beliefs derived from the first Research Question in terms of their planning and actual conduct of lessons in the classroom. Based on the findings of the Research Question 2, Research Question 3 called for an exploration of the alignment of teachers’ practices with the intentions of the new curriculum. Finally, the success in terms of implementation of the new curriculum would be explored in relation to Research Question 4. By answering these questions, significant findings could be advocated to the Ministry of Education for further action or improvement in learning of providing timely support to classroom teachers to help students gain relational understanding and learn mathematics meaningfully.

3.6 CHAPTER SUMMARY

This chapter has presented a review of literature in three areas: the nature and influence of teacher beliefs on student learning; constructivist theories and practices of mathematics education; and curriculum reforms and its challenges. In Bhutan, the recent introduction of the new curriculum required teachers to change their practices. Underpinning such changes in practice is a need for teachers to change their entrenched beliefs regarding mathematics and mathematics education. Anecdotal evidence, gathered by the researcher, has suggested that such change in practices and underlying beliefs is not widespread, and thus the aim of this study, embodied in its four Research Questions, was to investigate the alignment between teaching practices
and the intentions of the curriculum, and to form explanations for any potential misalignment. This was a significant undertaking as it explained theory regarding how, in the Bhutanese context, teacher beliefs shape classroom practices. The analytical framework was designed to support the analysis of teachers’ practices in relation to the proposed curriculum. In the following chapter, the research design that was applied to investigate the relationship between Bhutanese teachers’ beliefs and practices in the context of curriculum reform is presented.
Chapter 4: Research Design

This research investigates the beliefs and practices of Bhutanese primary teachers implementing a new mathematics curriculum as discussed in Chapter 1. This chapter presents the methodology adopted to seek answers to the following Research Questions:

Question 1: What are the beliefs of Bhutanese primary teachers about mathematics teaching?

Question 2: What are Bhutanese primary teachers’ planning and classroom practices in teaching mathematics?

Question 3: To what extent are mathematics teaching practices aligned with the curriculum intentions?

Question 4: What influences the implementation of the curriculum intentions?

To address these questions, it was necessary to choose a research methodology that would effectively capture the challenging work of the teachers and their relevant beliefs. A single embedded case study was chosen, with the case being teachers’ beliefs and practices in the context of the implementation of a new mathematics curriculum in Bhutan. It was conducted through an explanatory sequential mixed-method design in two different phases (referred to as macro and micro). According to Creswell and Clark (2011), the embedded case study is defined when the researcher combines the collection and analysis of both quantitative and qualitative data to examine a case. In this study, an explanatory sequential mixed method approach was adopted with a quantitative survey first followed by a qualitative method. The second, qualitative method was conducted to elucidate the initial results (Creswell & Clark, 2011).

This chapter outlines the details of the methodology and design of the study. Following the introduction, the researcher positions herself through a discussion of the research paradigm in Section 4.1. This is followed by Section 4.2, which discusses case study methodology. Section 4.3, provides an overview
of the study design. Section 4.4 describes the participants and the mixed-method approach, followed by methods adopted for data collection and analysis in Section 4.5. Issues of quality and ethics are discussed in Sections 4.6 and 4.7. Finally, Section 4.8 provides a chapter summary.

4.1 RESEARCH PARADIGM

Knowledge and understanding of the research paradigm are discussed in this chapter to determine how the study’s theoretical framework impacted on and informed the methodology. In general terms, a paradigm has been described as a set of basic assumptions (Creswell, 1998), including methodological, epistemological and ontological premises (Denzin, Lincoln, & Giardina, 2006). The understandings of these premises were key elements which placed the researcher within the research. The methodological, epistemological and ontological beliefs held by the researcher informed the theoretical framework in which this study was located. What follows is an exploration of these beliefs and the constructivist paradigm used in this study.

4.1.1 Constructivist paradigm

The embracing of a constructivist research paradigm for this study harmonised with the theoretical premises and knowledge of the researcher, who was conversant with key determinations, including that manifold realities are constructed by different individuals, and it is the understanding of their experiences that contributes to further knowledge and understanding (Merriam, 1998). The acknowledgement that reality is constructed socially, culturally, and historically (Lincoln & Guba, 2000) created an outline in which the researcher could begin to understand and describe manifold realities from the participants’ experience.

A key component of the constructivist research paradigm is that it adopts a relativist ontology which admits that multiple realities exist (Merriam, 1998). This supposition was a key component in determining that during the micro level phase, five participants would take part in the study instead of one, because each of the five participants would hold different views. At the same time, five participants was a manageable number for the timeframe of the micro level phase. The selection of participants for the research is explained further in
Section 4.5. A realistic set of methodological techniques was implemented so that data collection could occur in the participants’ natural settings (Denzin & Lincoln, 2011), that is within the participants’ schools and classrooms.

According to Bloomberg and Volpe (2008), any attempt to understand realities is context-specific and value-bound, rather than value-free. For this research, this has meant authenticating the role of the researcher, and the influence of the researcher on the research process. Lincoln and Guba (2000) described this process as multi-voice reconstruction, or that of being an active participant in the research. This acknowledgment of the researcher’s own impression on the research process provided an opportunity for placing the researcher within the research according to her own beliefs, values, culture, social, and historical experiences (Bloomberg & Volpe, 2008). This was been done in Chapter 1, Section 1.2, by discussing the researcher’s own history as an influence in this research.

Understanding of a constructivist model is derived from the assumption of meaningful exchange between participants’ experiences and the researcher’s interpretations of experiences. Merriam (1998) explains that “meaning is embedded in people’s experiences and this meaning is mediated through the investigator’s own perceptions” (p. 6). Hence, the relevance of this research was two-fold. First, placing the researcher in the research and acknowledging her contribution to the research was integral to the outcomes of this study. As the main instrument in the data collection, interpretation, and reporting of the cases, it was critical to monitor the impact of the researcher on the research process (Simons, 2009).

In this study, the researcher frequently explored and reflected on her role in the research process, including questioning how her own values and actions impacted on the research process (Simons, 2009). Second, this research aimed to build a deep understanding of the participants’ experiences in their own contexts. Documentation of the settings, experiences, and what was relevant to the teachers as recommended by Patton (2002) assisted in contributing to a holistic picture of the teachers’ experiences. Therefore, a constructivist paradigm was deemed the most appropriate to address the Research Questions in this study.
4.1.2 **The usefulness of a social constructivist lens**

Social constructivism was specifically used to understand and interpret teachers’ experiences in the current research (Vygotsky, 1978). In social constructivist research, it is acknowledged that individuals’ lives (in this case the teachers’) are constructed on a foundation of many truths, explanations, values, and beliefs that have resulted from histories and interactions with others (Patton, 2002). Teachers do not work in isolation; their working life is a complex set of interactions within complex organisations and cultures. Therefore, attention was paid in the research not only to their constructions of knowledge but also to how their knowledge had been influenced by wider socio-cultural viewpoints (Patton, 2002), both from their past and current teaching situations.

A social constructivist lens afforded the opportunity to acknowledge how people make sense of their world and are influenced by facets of their own culture and histories, and by time and place. It is important for teachers to realise that their local and national education contexts influence their experiences. The importance of a social constructivist orienting lens and the overarching constructivist paradigm were influential in all parts of the study, including the data collection, analysis and writing up of the findings.

4.2 **AN OVERVIEW OF CASE STUDY METHODOLOGY**

According to Glesne and Peshkin (1992), a case study is considered an appropriate methodology for a study that is qualitative in nature, as qualitative research is able to illuminate and gather insights about an issue, and describe manifold perceptions and realities, rather than focussing on quantitative results. Bassey (1999) pointed out that there are two major approaches to case studies. Robert Yin (1994) and Robert Stake (1995) are identified as the two main champions of case study methodology, and the two represent different methods of case study design and execution (Wainman, 2010). The arguments within case study research method mainly rest on the issue of the generalisation of the case study research findings, or how the study is transferable to other contexts. The present design uses a sequential explanatory mixed method (Creswell et al., 2003) under the umbrella of a single embedded case study implemented in two phases of analysis. As will be discussed, this approach is more closely aligned to
the ideas of Yin. Prior to discussing the adopted design, an overview of the two approaches to case studies is presented along with an alternate view presented by Thomas (2011). This provides as basis for introducing the specific form of case study adopted in this study.

4.2.1 **Robert Stake**

Robert Stake's (2005) approach to case study is interpretive, emphasizing particularisation not generalisation. The method focuses mainly on an individual case study which does not seek generalisability of results. According to him, three types of case study are: intrinsic, instrumental and collective. Stake argues that a case study concentrates on experiential knowledge of an event with close attention to the influences of its social, political and other contexts. Some features outside the boundaries of the case are also considered relevant (Stake, 2005). In this study, outside contextual features would include the historical circumstances of the Bhutanese education system and the new mathematics curriculum.

4.2.2 **Robert Yin**

Yin (2013) stated that “case study investigates a contemporary phenomenon in depth and within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident” (p. 18). According to his definition, one of the characteristics of case study research is the combination of a variety of data collection methods such as interviews, observation and document analysis. Yin’s approach to a case study emphasises the priori development of a hypothetical proposition to guide data collection and analysis. Based on his ideas, the approach in the current study has adopted an explanatory sequential mixed method case study approach.

The approach includes analytical generalisability where the study holds importance for others in a similar context (Wainman, 2010). That is, theories developed within one study to explain a phenomenon may be relevant in explaining similar phenomena in another situation. Yin also describes exploratory and descriptive case study approaches. Exploratory case study assumes that there are not pre-existing theoretical models. As the current study explores teachers’ beliefs and their alignment with the intention of the new
mathematics curriculum and its implementation in the classroom, generalisability was considered important. It was important to develop a theoretical perspective on the role of beliefs impacting on teaching in Bhutan. The purpose was to examine how Bhutanese teachers’ beliefs were reflected in their classroom practices and to determine whether their practices aligned with the theoretical intentions of the new mathematics curriculum.

Further, Yin (2013) argued that case study is an inquiry in a real-life context in a form of ethnography, focusing on a program or activity involving individuals. The case is described as a *bounded system*. In this study the case is bounded in terms of place by relating to a country, namely Bhutan, and selected teachers, and by focussing on the methods used to teach Class 5 students the concept of fractions.

### 4.2.3 Gary Thomas

According to Thomas (2011), a case study is defined as “a detailed discussion about some particular thing to make it complete and it is a kind of jigsaw puzzle to make a case complete and meaningful by trying to fit in the information from different angles” (p. 4). Thomas (2011) suggested that a case study should focus on one thing in depth, considering it from different angles, and thus a case study is not a single method but is instead a *wrapper* for different methods. Further, Thomas argued that a case can offer a rich picture with many kinds of insights coming from different angles, from different kinds of information such as interviews, observations, videos and so on.

### 4.3 OVERVIEW OF THE STUDY’S DESIGN

For the current study, the specific approach adopted was that of a single embedded case study based on Yin (2013)’s philosophy, with the case being teachers’ beliefs and practices subsequent to the introduction of a new curriculum in Bhutan. The study, as depicted in Figure 4.1, was implemented in two sequential phases, which are referred to as macro and micro level phases. The macro-level phase involved an analysis of beliefs of teachers selected from across Bhutan. Subsequently, the micro level phase was conducted in two schools and involved observations of the mathematics teaching practices of five
teachers. A mixed method approach to data collection was adopted, as presented in Figure 4.1.

As presented in Figure 4.1, the data for Phase 1 of the case study were collected through a survey distributed to 40 randomly selected primary schools across the country, with a total of approximately 120 surveys distributed. Of the 120, 80 questionnaires were returned for data analysis. Data were analysed using simple descriptive statistics. In the micro level phase, data were collected mainly through lesson observations of five classroom teachers as they taught a unit on fractions in two primary schools (Takin & Dragon). Observational data were supplemented with data from interviews and lesson plans. All five participating teachers taught Class 5 students in four different sections (two sections of Class 5 per school). The background details for adopting this mixed method design are presented in the following section.
4.4 MIXED METHODS APPROACH

As argued by Johnson and Onwuegbuzie (2004), what works in research is what should be used, regardless of any philosophical or paradigmatic assumptions. Researchers are obliged to consider the notion of aligning for purpose when deciding on the methodological approach to be taken (Hesse-Biber, 2010). Researchers have taken varied stances on integrating paradigms into mixed method approaches, recognising that each paradigm and their combined use must contribute to new understandings (Creswell & Clark, 2011; Greene & Caracelli, 2011). Mixed methods research has been defined in a number of ways, but in general, it comprises both quantitative and qualitative research measures in a single study. As reported by Ivankova, Creswell, and Stick (2006) when these data (quantitative and qualitative) are combined, there is more potential for triangulation and more rigorous analysis. Thus, Creswell (2008), defined mixed method as “a procedure for collecting, analysing, and integrating both quantitative and qualitative data at some stage of the research process within a single study for the purpose of gaining a better understanding of the research problem” (p. 3).

Further, mixed method research is recognised as one of the major research approaches currently used (Bryman, 2007; Doyle, Brady, & Byrne, 2009; Johnson & Onwuegbuzie, 2004; Sandelowski, 2000; Tashakkori & Creswell, 2007; Yin, 2006). Of the 40 different mixed methods designs reported in the literature (Teddle & Tashakkori, 2003), Creswell et al. (2003) identified the six most frequently used designs. These comprised three concurrent and three sequential designs. Of the sequential designs, the mixed method explanatory sequential design is standard among researchers, and involves collecting and analysing first quantitative and then qualitative data in two consecutives phases within one study (Creswell & Clark, 2011; Ivankova et al., 2006).

The design starts with the collection and analysis of quantitative data. Phase 1 (quantitative) is followed by the subsequent collection and analysis of qualitative data. Phase 2 (qualitative) is designed so that it follows from the results of phase 1. This design was adopted in this study to seek baseline through survey questionnaires followed by deeper exploration using qualitative methods as shown in Figure 4.2.
As proposed by Ivankova et al. (2006), in such mixed methods design, the order of the quantitative and qualitative data collection is determined by the study purpose and research questions. When a quantitative phase is placed first in the sequence, the quantitative results guide the in-depth qualitative phase which follows. In this study, the Phase 1 (macro level) used a survey questionnaire to ascertain teachers’ beliefs about mathematics teaching and their existing practices. This phase mainly explored the first three Research Questions. Phase 2 (micro level) was designed to explain in greater depth and expand the results obtained in Phase 1. This was done primarily through observation of lessons to see whether there was an alignment of teachers’ practices with the beliefs about mathematics and mathematics teaching as revealed in Phase 1. The unit of analysis was the classroom practice of a group of Class 5 teachers. One of the most difficult challenges for mixed methods researchers is how to analyse data collected from quantitative and qualitative research. According to Bryman (2007), researchers do not always bring their findings together and tend to miscue the potential of each method to offer
insights, or help clarify each other. The specific methods adopted are discussed in the following section.

4.5 METHODS

In this section the methods of the study including participants, data sources and data analysis for each of the two phases are described in detail.

4.5.1 Phase 1 (Macro level)

Since research of this type has not been done before in Bhutan, the macro level study was designed to explore general information regarding Bhutanese primary mathematics teachers’ beliefs about mathematics and mathematics teaching. The process was expected to reveal in general terms, the alignment of the intentions and implementation of the new mathematics curriculum, which was to be explored further in-depth during the micro level phase. The participants in this phase were teachers from 40 randomly selected primary schools from 20 districts (two schools from each district) teaching mathematics to Classes PP – 6.

According to Miles, Huberman, and Saldana (2014), random sampling is considered the gold standard of quantitative research in terms of good coverage of data. Three teachers from each of the 40 randomly selected schools were expected to complete and return the questionnaires to provide a total of 120 respondents. Surveys were distributed to generalist primary school teachers, as there is no dedicated teacher allotted to teach mathematics in Bhutanese primary schools. A set of three questionnaires was posted through the address of the respective principals, who in turn distributed them to those teachers teaching mathematics in the school. This total represented a ratio of 1:20 between expected respondents and the total population of primary school teachers in the country. The response rate of the returned questionnaires was 67% of the total distributed (80 out of 120).

Survey Construction

Creswell and Clark (2012) have defined survey designs as “procedures in quantitative research in which investigators administer a survey to a sample or a population of people to determine the attitudes, opinions, behaviours, or characteristics of the population” (p. 376). Creswell and Clark (2012) consider survey tools to be very effective in helping to identify views of individuals.
Survey tools are also found to be efficient and enable straightforward data processing, especially by using modern computer software. Moreover, Dornyei (2007) stressed the advantage of reducing the amount of time required by respondents to express their opinions while responding to survey questionnaires. Hence, a survey was considered as one of the most suitable instruments for use by both the respondents and also for the researcher, in terms of limited time and resources.

Hinkin (1995) suggested that since “keeping a measure short is an effective means of minimizing response biases caused by boredom or fatigue, at least four items per scale are needed to test the homogeneity of items within each latent construct” (p. 109). The survey was based upon the work of Perry et al. (1999), aligned with Hinkin (1995)’s suggestion, with the exception of a few items, which were added to suit the context of the current study. The survey questionnaire comprised six categories; the details are given in Table 4.1. and the full survey provided in Appendix B.

Table 4.1
Summary of Survey Categories

<table>
<thead>
<tr>
<th>Category</th>
<th>Content</th>
<th>Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Respondents’ demographic information</td>
<td>8</td>
</tr>
<tr>
<td>II</td>
<td>Respondents’ beliefs about nature of mathematics</td>
<td>6</td>
</tr>
<tr>
<td>III</td>
<td>Respondents’ beliefs about mathematics learning</td>
<td>7</td>
</tr>
<tr>
<td>IV</td>
<td>Respondents’ beliefs about mathematics teaching</td>
<td>8</td>
</tr>
<tr>
<td>V</td>
<td>Respondents’ attitude towards new mathematics curriculum</td>
<td>6</td>
</tr>
<tr>
<td>VI</td>
<td>Designing of sample learning activity</td>
<td>1</td>
</tr>
</tbody>
</table>

The first category was about respondents’ demographic information, specifically gender, age, qualification, teaching experience, position, location, class size, and class level taught. Categories II-IV comprised a total of 21 items, which were used to explore respondents’ beliefs about mathematics and its teaching and learning. These items were based on the work of Perry et al. (1999), and Perry, Way, Southwell, and White (2005). These researchers investigated three aspects of teachers’ beliefs: the nature of mathematics; mathematics learning; and mathematics teaching. The authors reported the instrument identified two factors: child-centeredness and transmission. Those
factors closely represented the two different types of beliefs (experimentalist and instrumental beliefs) categorised by Ernest (1989).

Of these two, the intentions of the new curriculum are aligned closely with the philosophical views adopted by experimentalists. Hence, those belief tools adapted from Perry et al. (1999) were found appropriate for this study. Category V comprised questions that probed respondents’ beliefs and acceptance of the concepts of the new mathematics curriculum. The six items were designed by the researcher to explore respondents’ awareness on the intentions of the new Bhutanese curriculum.

A Likert-type scoring format was used, and participants were asked to indicate the extent to which they agreed (or disagreed) with each statement presented. These types of scales are considered to be the most useful in behavioural research. According to Hinkin (1995), the use of Likert-scales is argued to help avoid loading participants with work, and certify an overall view of the focus of the research. Likert-type scales can vary in the number of scale points (e.g., 4 or 7 points), as well as in the descriptors. A five point scale was adopted, as it is widely recognised as a proxy interval level of measurement in line with common practice in educational research (Hinkin, 1995). A score of 1 was assigned to the Strongly Dis-agree (SD) response and a score of 5 to the Strongly Agree (SA) response.

In Category VI, a space was provided for participants to design a short learning activity for one of a list of topics provided appropriate for different class levels (Classes Pre-Primary to 6) as a sample of their approach to teaching mathematics. This was to gain insights into the alignment of respondents’ beliefs with what they practise in their classrooms.

To ascertain any problems relating to the use of terminology in the survey that might be confusing for teachers, the survey items were piloted informally with six general primary school teachers and one secondary mathematics teacher residing in and around the residence of the researcher. Items were well understood, so the pilot did not reveal any obvious bias effects (Dornyei, 2010; Saris & Gallhofer, 2007). The time needed for this process from pilot study to completion of the survey was initially assigned a period of four weeks; however,
in reality, it took an entire academic semester (January to June 2013), because some schools were in very remote locations, which delayed the mail.

**Survey administration**

Data collection procedures started with the distribution of 120 survey questionnaires to 40 randomly selected primary schools across the country. Selection of schools was done based upon the 20 school districts (or Dzongkhag). A list of all the primary schools in Bhutan was requested from the Ministry of Education. Schools were identified by randomly selecting names from a set of 20 containers in which schools from each Dzongkhag were represented. Each container contained the names of all the primary schools in that Dzongkhag. Two rounds of picking randomly from the containers (one strip at a time) resulted in the selection of 40 schools that were invited to participate in the study.

The researcher used the postal service to distribute questionnaires to the 40 randomly selected schools across the country, roughly representing the school settings of both urban and rural regions. According to Saris and Gallhofer (2007), many researchers prefer using mailed or electronic surveys (internet-based surveys) because of their speed, accessibility and a high response rate. However, emails or internet-based surveys were not ideal for the current study for numerous reasons. For instance, most current teachers have been taught by the researcher, or are at least known to her, so an email address would be likely to eliminate anonymity, or inhibit honest responses. Therefore, the researcher decided to distribute through postal mail, addressed to the respective principals of the schools. The respective principals then distributed to any three teachers teaching mathematics in the school to participate. Beside three questionnaires, each packet included a covering letter to the principal along with a copy of an office order from the Ministry of Education granting permission for the school to take part in the research. The packet also included a return envelope.

**Survey analysis**

As indicated in Figure 4.3, data analysis for macro-level phase was undertaken in two parts: The first part of the questionnaires contained demographics and belief statements and the second part was composed of sample learning
activities. The method used for each of the part is described in the following sections.

**Demographic and belief statements analysis**

As presented in Table 4.1, the first category was about the respondents’ demographic and the second, third, fourth were about belief statements concerned with the nature of mathematics, mathematics learning and mathematics teaching. The fifth category was a beliefs statement associated with the new mathematics curriculum. Data from the first to fifth categories were analysed item-wise using SPSS software, as indicated in Figure 4.3.

![Figure 4.3. Data analysis for macro-level study.](image)

A separate table was drawn for each of the categories (respondents’ demographics, nature of mathematics, mathematics learning, mathematics teaching and new curriculum) to present information about frequency of data, mean, standard deviation and percentages. Beside this, cross-tab analyses were also carried out within the category of demographic information, such as respondents’ qualifications, teaching experience, and geographical location, and also geographical locations and class size. The cross-tab analysis was intended to explore whether there was any influence of teachers’ qualifications on their teaching experiences and geographical location; similarly with the impact of class size by geographical location.

The reliability (internal consistency) of the Likert-scale survey and its subscales was calculated by using SPSS. Analysis using Cronbach did not demonstrate the level of reliability reported by Perry et al. (1999). Therefore, analysis question by question was undertaken. However, the use of mean alone turned out to be an inappropriate measure of central tendency, as responses to
many of the items were not normally distributed; secondly, the scale was ordinal
not interval. As a general rule of thumb, when skewness or kurtosis statistics fall
outside a boundary of 1 or -1, there could be problems in interpreting any
inferential statistics. Given this type of data, the median was found to be a better
statistic to report. Moreover, in terms of the Interquartile Range (IQR), the
smaller the IQR, the more consistent was the response; the larger the IQR, the
wider is the range of responses. The IQR in most of the responses was small,
suggesting a degree of uniformity in the responses. Using all these tools, a
background picture was painted in terms of teachers’ beliefs and their classroom
practice, which was explored further in the micro level phase of the study.

Sample task category analysis

As introduced earlier, the sixth category of the survey was to design a
sample learning activity for the chosen level on fractions as shown in Table 4.1.
Each respondent was asked to design a sample learning activity as discussed
earlier in this chapter. Since this category provide qualitative data, it was not
possible to enter it in the SPSS software like the first five categories. Data from
this category was coded and analysed using two different approaches, the (3+1)
analytical rubrics and frequency analysis of process standards. In addition to
using the 3+1 analytical framework, the frequency analysis of process standards
was carried out to explore the extent to which teachers provided opportunities
for students to learn by doing.

A framework containing the five NCTM process standards was used for
coding and analysing the implementation of curriculum intentions in designing a
sample learning activity. Likewise, using the 3+1 frameworks as presented in
Sections 3.5.1–3.5.3, a processes used for coding are as follows. The researcher:

- Mapped 1st ten samples against the 3 + 1 framework;
- Took notes on the mapping done with the 1st ten samples;
- Cross checked the reliability of the coding done on the 1st ten samples
- Mapped the remaining 62 samples against the same but with an
improved version of the framework.

To begin, the researcher piloted the coding with the first 10 samples against the
framework. Those ten samples were also coded using the same framework by
the researcher’s team of supervisors to confirm the reliability of the coding procedure adopted. Following discussion and clarification of codes, an improved version of the coding framework, which provided consistent coding, was developed, and the remaining 62 samples were coded.

Table 4.2  
**Coding Procedure for Sample Learning Activities**

<table>
<thead>
<tr>
<th>3+1 Framework</th>
<th>NCTM Process Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scanned out all the sample learning activities from the main body of the questionnaires</td>
<td>Prepared a sheet of paper presented vertically with five NCTM process standards</td>
</tr>
<tr>
<td>Numbered each of the sample learning activities based on the order of questionnaire received by the researcher in sequence</td>
<td>One sample at a time, ticked against each when found a match from the sample learning activity and continued with the rest of the samples</td>
</tr>
<tr>
<td>Prepared three sheets (A3) paper for the three curriculum intentions with appropriate rubrics, as indicated in the respective 3+1 framework</td>
<td>Counted the number of ticks against each of the standards and converted into percentage</td>
</tr>
<tr>
<td>One sample at a time, indicated with sample name wherever appropriate on those three sheets (e.g. Item 1 written wherever matched)</td>
<td>Of the five, three standards such as communication, connection and representation were further sorted into sub-sections and matched against all the samples.</td>
</tr>
<tr>
<td>Counted the number of sample name reflected on every sheet of the papers (intentions) and converted into percentage based on the total number of samples (i.e. 72)</td>
<td>Each sub-section was further counted and converted to percentage</td>
</tr>
</tbody>
</table>

To reduce overlap and redundancy of codes, the researcher double-checked by taking the list of codes and going back to the data, and circling or high lightening the quotations that supported the codes. The same procedures of using 3+1 framework was used for coding data collected under the micro-phase level study, which is presented in the following section.

4.5.2 **Phase 2 (Micro-level)**

Phase 2 of the research aimed to explore in more depth teachers’ beliefs and practices about mathematics and mathematics education and their alignment with the intentions of the new curriculum. For this, research tools used in collecting qualitative data comprised of video-taped classroom observations, audio-recorded informal semi-structured interviews, teachers’ lesson plans and reflections, and the researcher’s field notes. According to Creswell and Clark (2012), the basic principle of qualitative research study is that “qualitative study
relies more on the views of the participants in the study and less on the direction identified in the literature by the researchers” (p. 17). Aligning with this statement, the details of the qualitative method adopted for this study are presented in the following sections.

The context

Two primary schools (Takin and Dragon) were selected to provide in-depth understanding of the findings (teachers’ beliefs and practices) derived from the macro level phase study through contextually distinct settings, namely urban and rural, and to gather evidence related to the four curriculum intentions and the teachers’ implementation of the new curriculum as practised in classrooms. The two schools were selected because they had two to three mathematics teachers teaching each of Classes PP to 6. Further, both the schools were coincidently members of the 40 randomly selected schools to which survey questionnaires were administered. The reason for choosing Class 5 was because this level falls in the middle of Stage 1 (PP-3) and Stage 2 (4-5). Students at this class level have just completed lower primary level education and entered into upper primary level schooling; it is a transitional period from lower to upper primary education, and it allows a focus on the quality of mathematics education given to young students.

As pointed out in the study conducted by Fillingim (2010), the elementary mathematics experience is a crucial time for students, during which they establish a foundation of knowledge, skills and beliefs regarding their ability to succeed in mathematics. Hetherington (2001) argued that if students do not begin to transfer knowledge during their early years, they will not have the core skills and knowledge needed to succeed in mathematics at the middle and high school levels. Moreover, according to Fillingim (2010), although similar research has investigated higher-level secondary students, very few have focussed on younger students and elementary-level students, despite the significance of these years to engagement in mathematics.

The reason for choosing fractions was that the topic is not popular among Bhutanese primary mathematics teachers and, from the researcher’s experiences, is considered by them to be very difficult.
Participants

Five teachers were chosen based on their interest in participation. This small number of participants was considered manageable to explore teachers’ beliefs about mathematics and mathematics education, with three teachers from Takin and two from Dragon. The organisation of participants is summarised in Figure 4.4.

![Figure 4.4. Participants in micro level study.](image)

The teachers selected were required to have a minimum of five years of teaching in primary schools as such teachers were expected to be able to compare the old and new mathematics curricula. As described earlier, the new curriculum for Class 5 was introduced in 2008. The next criterion for inclusion was a teacher with a high interest in teaching mathematics, with the expectation of a sound mathematical knowledge; such teachers would be eager to learn more about handling the new mathematics curriculum.

For the purpose of the current research, the five participants were given pseudonyms: Phurba, Norbu, Sangay, Gawa and Tempa. They were all from different backgrounds in terms of gender and qualifications (PTC, B.Ed Primary Maths & B.Ed Secondary Maths), as presented in Table 4.3.
Table 4.3
Demographics of Case Study Participants

<table>
<thead>
<tr>
<th>School</th>
<th>Teacher</th>
<th>Gender</th>
<th>Qualification</th>
<th>Teaching experience (years)</th>
<th>Year level taught</th>
<th>No. of students in observed class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Takin</td>
<td>Phurba</td>
<td>Male</td>
<td>B.Ed</td>
<td>7</td>
<td>5</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Primary Maths</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Norbu</td>
<td>Female</td>
<td></td>
<td>B.Ed</td>
<td>8</td>
<td>5</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Secondary Maths</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dragon</td>
<td>Sangay</td>
<td>Female</td>
<td>B.Ed</td>
<td>8</td>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Secondary Maths</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gawa</td>
<td>Male</td>
<td></td>
<td>PTC</td>
<td>13</td>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>Tempa</td>
<td>Male</td>
<td></td>
<td>PTC</td>
<td>13</td>
<td>5</td>
<td>40</td>
</tr>
</tbody>
</table>

Note: PTC (Primary Teacher Certificate)

Data sources

To develop an in-depth understanding of the chosen case, any study requires the collection of multiple forms of data (Creswell, 2008). Consistent with case study design as described by Yin (2013), there are six sources of evidence to be considered and implemented: documentation; archival records; interviews; direct observations; participant observations; and physical artefacts. Of these, four sources were included in this case study: documentation (i.e. policy documents, the teacher’s guidebook, students’ textbooks and examination papers); interviews of teacher participants; observation of mathematics lessons; and physical artefacts, namely teachers’ lesson plans and reflections, students’ sample work, and researcher’s field notes. Specific details of these data sources are discussed in the following sub-sections.

Documentation

Three important documents included in this study were policy documents, the teachers’ guidebook, students’ textbooks, and past examination papers. Respecting the ideas that changing practice is difficult, the introduction of the new curriculum provided materials such as textbooks for the students and guidebook for teachers. The newly introduced textbook and its guidebook were
expected to be used by teachers as reference to keep a track of the implementation of the new mathematics curriculum.

Besides the use of the textbook and the teachers’ guidebook, the researcher examined the previous examination papers (between 2007 and 2013), in order to ascertain whether Class 5 level questions focus on process or product, thereby revealing the depth of alignment with the intentions of the new curriculum. The purpose was to compare the type of instruction provided to the type of questions asked in the examination and the type of result achieved. In this way, information on the implementation of the new mathematics curriculum was expected to be revealed.

Lesson observations

According to Brikci and Greene (2007), observation by direct participation is considered to be one of the best research methods, as it helps the researcher to understand fully the complexities of situations. Prior to the actual video lesson recording, the researcher conducted one mock session (O’Brien, 1993) of the videotaping procedure in each class. This allowed teachers to view themselves teaching before the actual data collection, to become familiar with having the camera in the classroom, and with the process of being observed. In the days leading up to the data collection, students in the selected classes also needed to become familiar with the presence of the researcher in the class. The researcher found that the mock lesson was also an opportunity to become familiar with setting up and operating the equipment in the classroom as a way to avoid potential technical problems during data collection. Data from the mock session were not used in the study.

As suggested by O’Brien (1993), for the current research, two cameras were set up in the classroom, one to video the teacher and any other major instructional resources (such as slides and blackboard), and the second to video the students and their activities. Camera one was positioned at the front of the class where the teacher generally stands to teach the lesson. This placement allowed the researcher to capture the actions of the teacher as none of the participants moved far away from this position. Camera two was positioned at the back of the class, which allowed the researcher to capture the whole of class
dynamics during class time. In total, the current study included 28 video recorded lessons.

Participating teachers were observed teaching the whole unit on fractions for Class 5 level with 40 students, which took a minimum of six weeks to a maximum of eight weeks in each of the two schools. Each observation was for a full class period. However, the total number of lessons taken to complete teaching a Unit (Fractions) in Class 5 differed. Consequently, the number of lessons observed was 16 lessons from Takin (eight lessons in each class) and 12 lessons from Dragon (six lessons in each class). Detailed notes were taken during the observations to provide a close picture of teacher and student/s participation aligning with the intentions of the new curriculum. Similar to the study conducted by Sztajn (2003) on adapting reform ideas in different mathematics classrooms, in this study, teachers’ beliefs were anchored strongly on the observation of their classroom practices and their lesson reflections.

**Interview**

Interview is considered as one of the most powerful means to understand our fellow human beings and it can take a variety of forms (Fontana & Frey, 2000). Kvale and Brinkmann (2009) defined the interview as an interchange of views between the interviewer and interviewee(s) conversing about a theme of common interest. Similarly, Seidman (2006) stated that, “the root of in-depth interviewing is an interest in understanding the lived experience of other people and the meaning they make of that experience” (p. 9). Interviewing is considered a rich data collection technique and a main means of understanding peoples’ beliefs in greater depth. Yin (2012) argued that one of the most important bases of case study information is the interview. Therefore, interviewing was used in the current research to collect data to determine teachers’ beliefs about mathematics and mathematics education in terms of its alignment with the intentions of the new curriculum.

Informal post-lesson interviews were conducted in a group comprising of two to three members after every lesson, to hear to teachers’ lesson reflections related to their beliefs and what they practised in the classroom. Participants were asked to share information about their practices regarding the implementation of the curriculum intentions in their class. This kind of
The interviewing method was chosen as it permitted teachers to expand on their beliefs after observing these beliefs enacted in their teaching practices. The interview protocol questions for the first introductory post-lesson interview are listed below:

- How do you define the term mathematics? Why?
- What strategy do you use to encourage students in gaining deeper understanding of mathematics? Why?
- What is your opinion about ‘Emphasise more on why something is true and not simply that it is true’?

The questions during the rest of the post-lesson interview were adopted to reflect the type of lesson conducted and lesson reflection provided by the participants. The interviews were audio-taped and transcribed for analysis as data for the study. The format of the interview, while addressing particular questions, was informal and conducted as a conversation. The researcher acted as a moderator with general questions, acquiring responses from the teacher participants. In this way, the researcher encouraged participants to listen to each other and helped them to develop their ideas more clearly or to articulate new ideas.

The nature of discussion during the post-lesson interview was quite similar to a prolonged interview, which is referred to as a type of ongoing discussion different from formal or semi-structured interviews (Sikes, Measor, & Woods, 1985), where specific questions or themes are not pursued. The discussions are said to be unrestricted and take place over a longer period of time. This method of data collection allowed a more holistic understanding of the teachers’ insights, providing a greater depth of response.

Communications were kept informal so that the teachers felt comfortable to discuss their lessons, to create a friendly atmosphere and avoid the impression that they were being assessed and needed to justify their practices. A professional relationship already existed between most of the participating teachers and the researcher, because most graduated from the College at which researcher has worked for the last 18 years. This also meant that the communication could be relatively informal from the beginning of the study and
that the teachers felt comfortable communicating in a reflective manner with the researcher, although it is acknowledged that a power relationship did exist (see Section 4.7). Besides face-to-face conversation, ongoing reflection between the teachers and the researcher was negotiated at the beginning of the data collection. Hence, email correspondence continued after the observation period. Email was chosen for its convenience and its facility to preserve a record of the reflections. However, due to the teachers’ time constraints, correspondence through email became rare.

*Physical artefacts*

The physical artefacts included a copy of all the lesson plans taught on ‘fractions’ in Class 5 during the data collection period, along with a set of sample learning activities given to the students, and the researcher’s daily reflection diary of the research activities. A reflection diary was maintained by both researcher and participants. The main purpose of collecting such physical artefacts was to study the alignment of the intentions and implementation of the new curriculum. Similarly, the purpose of collecting students’ sample work was to compare these with the expectations of the lesson plans aligning with the intentions of the new curriculum. Such sample work was expected to prompt teachers to reflect on and plan accordingly for the next lesson. Moreover, such artefacts had the potential to reveal the quality of students’ understanding. Participants used a diary to reflect on their lesson practices, whereas the researchers’ diary included a record of all the activities carried out during the data collection time, including written notes of the lessons taught. The researcher maintained a diary of field notes to strengthen the data collected.

According to Yin (2012), note-taking of some sort is common to virtually every case study. Yin also argued that field notes can be taken from different sources of evidence, including open-ended interviews, document review, or observations that have been made in a field setting (Yin, 2012). Creswell (2005) stated that field notes are “the data recorded during an observation” (p. 213) and suggested two types of field notes both of which were used for the current study: “descriptive field notes and reflective field notes” (p. 214). Descriptive field notes were taken to provide a description of the events, activities, and people involved in the conduct of lessons. These notes were considered when watching
the video recording to see if there were any discrepancies between what the researcher thought she had seen, and what the video recording revealed.

Reflective field notes were also taken after the recorded teaching sessions. These recorded the researcher’s personal thoughts during the lesson observations. For the current research, the researcher took notes during the lesson observations and also after the teaching session. These sources of evidence were used to support the analysis of other data. At the end of each lesson, a post-lesson debriefing took place; its duration depended on the availability of the teachers’ and ranged from 30 to 60 minutes. The debriefing was intended to discuss and reflect on the teachers’ strategies, and provide feedback on the design of the instructional program. All the details of face-to-face discussions and the researcher’s own reflections were recorded in the researcher’s reflection diary. The reflection diary proved very useful at the time of data analysis.

**Data analysis**

The data analysis procedure was carried out according to the structure given in Figure 4.5.

![Figure 4.5. Data analysis procedure for micro level study](image)

Twenty-eight classroom observations were analysed using the 3+1 framework. The other data sources such as teachers’ lesson plans, teachers’ lesson reflections, and teachers’ post lesson interviews, were analysed using a
frequency analysis based on four lesson components (exploration, exposition, formative assessment, and practice and applying as suggested by the new curriculum. In the following sections, brief details of each of the data handling procedures adopted, such as data transcriptions, data coding and data analysis, are presented.

**Transcribing Data**

Kvale (2007) argued that the amount and form of transcribing depends on factors such as the nature of the materials and the purpose of the investigation, the time and money available, and the availability of a reliable and patient typist. In the current research, as a PhD student with time and resource limitations, the researcher transcribed in English, her own 14 post lesson interviews and 28 video-recorded lessons prior to the data analysis to explore answers to the respective Research Questions. Since English is used as a medium of instruction in Bhutanese schools, both interviews and lessons were conducted in English. The transcription process also involved listening to the interview tapes several times, during which notes were taken about the tone of voice used by participants when describing their experiences, pauses in conversation, and emphasis on certain points which were important to the participants. These notes were taken as future reference points during the process of analysis.

**Data coding**

Creswell (2012) argued that there is a large amount of literature on qualitative data analysis, with many different viewpoints about the process. According to Gibbs (2002), coding depends on how the investigator defines the data. It is likely that although there exist different terms to refer to the processes of data analysis, they are similar in the techniques of analysis. For this research, the processes of coding and categorising used mainly paper Creswell and Clark (2012). The researcher concluded the procedures manually with the help of colour coding.

Gibbs (2002) argued for two ways of coding, concept-driven and data-driven methods. Concept-driven coding is considered a method that builds up a list of thematic ideas based on key words from the literature review, previous studies, or topics in the interviews and then data are coded using the list. The researcher analysed data from both the video-recorded classroom observations.
and their associated lesson plans to understand teachers’ actual teaching practices and their alignment with the intention of the new curriculum. To enact this, the researcher used the same coding procedure as presented for analysing sample learning activities under macro-level phase. The 3+1 analytical framework was developed based on the four intentions of the reformed curriculum and these were mapped onto the related data.

**Procedure for coding 28 video-recorded lessons**

The procedure for coding the 28 video recorded lessons was the same as coding the 72 sample learning activities using the 3+1 framework as presented earlier. In addition, frequency analysis was used by colour coding of the 28 lesson plans and observations according to the four suggested lesson components (exploration, exposition, formative assessment and practice and applying). Against each of the lesson plans and video recordings provided by the five teachers, the suggested lesson components were colour coded to reveal the extent of alignment between practice and planning. The same frequency analysis coding procedure was used for the interviews transcripts. The whole process for each lesson plan and lesson observations transcribed was done manually with the help of colour coding to map the information according to the analytical framework (3 + 1).

**Data analysis**

This study adopted a descriptive approach to produce a narrative of the alignment between the intention and implementation of the new curriculum. The main purpose of adopting this analysis approach was to explore the common issues that help determine the extent of alignment between the intention and implementation of the curriculum. Based on the data, accounts of the developing experiences of the teachers were analysed. The findings and evidence from all the data sources were used to explain teachers’ actions and any apparent changes. These notes were written in the form of discussion between the teachers and the researcher, as stated by Creswell (2012): “the primary form for representing and reporting findings in qualitative research is a narrative discussion” (p. 274). No particular format was used in writing texts as there was no set form for these narrative discussions (Coffey & Atkinson, 1996). Direct
quotations from the written communications connected to noteworthy events that indicate the evidence of current practice are included in the discussion chapter.

4.6 STUDY QUALITY

To ensure the rigour and trustworthiness of a study, Yin (2012) suggests researchers apply different strategies at different research stages. In this current study, the following strategies were used to maintain rigour in the process.

4.6.1 Pilot testing

To ensure reliability of the survey questionnaires and analytical rubrics of the 3+1 framework, pilot testing was conducted prior to the actual application. Pilot testing was conducted to cross check the feasibility of the chosen tools when used for actual application. For instance, survey questionnaires were pilot tested informally with a group of teachers residing nearby the researcher’s residence. Similarly, a group of research students with mathematics and science backgrounds were invited to analyse a set of 10 sample learning activities using the 3 + 1 framework. Each participant was provided with a set of ten randomly selected sample learning activities along with a sheet of process standard framework. Each pilot participant coded the tasks in terms of the occurrence of each process standard. This pilot test allowed the researcher to confirm and refine her interpretation of each process standard. Moreover, this pilot analysis was done to check the validity of the framework in terms of mapping the content of sample learning activities aligning with the curriculum intentions. Further, the team of research supervisors was invited by the researcher to evaluate the 3+1 framework with an identical set of sample learning activities. There were very minor changes to be incorporated prior to its actual application.

4.6.2 Maintenance of anonymity

To ensure credibility and avoid bias, respondents for the survey questionnaires were anonymous. Data collected were thus expected to reflect participants’ honest views. The same principle was applied to the two selected schools and five participating teachers in terms of using pseudonym names to protect their identity.
4.6.3 Long-term engagement

According to Gibbert and Ruigrok (2010), conformability refers to the extent to which an investigation procedure leads to an accurate observation of reality. Aligning with this idea, lesson observations did not stop at one or two lessons but continued to the completion of the teaching of a whole unit on fractions in Class 5. To complete a whole unit, it took six to eight lessons taught by each of the participants. Hence, chances of duplication and artificiality of the situation were reduced, and reliability and conformability of the data analysis were increased.

4.6.4 Video/audio-recorded lessons and interviews

In order to maintain credibility of the data, all observed lessons and teacher interviews were video- and audio-recorded for further reference and cross-checking purposes. In this way, identification of any mistakes or inconsistencies could be easily detected; this improved the quality of findings by increasing the reliability of the interpretation of the results. Moreover, video-recorded data can be re-used in the form of a video clip to enhance the justification of certain findings. Similarly, audio can be replayed to extract evidence and improve the quality of findings.

With reference to Denzin and Lincoln (2000) the researcher’s application of triangulation in the process of data collection is presented in the following paragraph. To add authenticity of the video recording of the lessons and the reality of the actual classroom, during the data gathering period, some correspondence took place between the researcher and her supervisory team in regard to the data gathering process and the quality of the data gathered. In addition to this, the principal supervisor of the researcher visited the researcher in Bhutan and visited one of the classes that were typical of the classes from which data (i.e., video recording) were gathered. Further, the researcher and her supervisory team worked closely together, including assistance received from other experts from QUT, particularly in terms of interpreting the statistical analysis and the piloting of the 3+1 analytical framework to code the work samples. Together, these are all examples of the approaches to ensuring the credibility and reliability of the data gathering and its analysis.
4.6.5 **Trustworthiness**

Besides making the survey anonymous, the researcher attempted to maintain the trust of the participating teachers. In order to develop their confidence and to derive factual data, the researcher intentionally made the research purpose clear during the pre-visit to the schools. Her emphasis was on sharing a common goal of improving the teaching and learning of mathematics. The researcher also reminded participating teachers not to consider her someone trying to evaluate their practice. Therefore, they should feel free to express anything related to teaching and learning of mathematics.

Given Bhutanese cultural and social characteristics such as respect for teachers and people in positions of authority, it was perhaps inevitable that the interviewees might give responses that they thought the researcher wanted rather than their true feelings (Atkinson & Silverman, 1997; Fontana & Frey, 2000; Silverman, 2013). However, the researcher believes that trustworthiness of her data is evidenced through:

- Interviewees honestly responded to the questions asked by the researcher about why they did not use the *Try This* activity, saying that it took too much time and the old curriculum was easier to teach because students just had to memorise the formula and not think;
- The data from the survey of the much larger group of 80 teachers strongly indicated that although they endorsed the new curriculum and its intentions, their sample activities still reflected the reliance on older pedagogical approaches.

4.6.6 **Triangulation of findings**

According to Denzin (1978), “triangulation is broadly defined as "the combination of methodologies in the study of the same phenomenon” (p. 291). Similarly, Morse (1991) stated the purpose of triangulation is “to obtain different but complementary data on the same topic” (p. 122). According to Creswell and Clark (2011), triangulation is used when a researcher wants to match and compare quantitative statistical results with qualitative findings or to validate these. Hence, in this study, triangulation of findings was possible using five
sources of data (survey questionnaires, sample learning activities, lesson plans, lesson observations, and interviews).

4.7 ETHICS

Approval from the Queensland University of Technology (QUT), University Human Research Ethics Committee (HREC) was obtained prior to the commencement of the study, with the code number 1300000027. Data collection was conducted in Bhutan with teachers teaching mathematics in 40 randomly selected primary schools for macro phase and micro at two chosen schools selected for their appropriateness to the research design. Further, approval from the concerned authorities in Bhutan to conduct this research was obtained. Full details of the ethics package can be found in Appendix A. A participant information sheet and consent form in English was provided to participants.

4.8 CHAPTER SUMMARY

This chapter has presented the research design and provided the rationale for the choice of participants, methods, data collection procedures and the data analysis processes employed in the current study. The details are summarised in a matrix showing the research questions, data sources and the methods adopted are presented in the Table 4.4.

Table 4.4

<table>
<thead>
<tr>
<th>Research Questions</th>
<th>Data Source</th>
<th>Data Collection Tool</th>
<th>Data Analysis Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>What are the beliefs of Bhutanese mathematics primary teachers about mathematics</td>
<td>80 Primary teachers (respondents)</td>
<td>Questionnaires</td>
<td>Basic descriptive statistics</td>
</tr>
<tr>
<td>teaching?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>What are Bhutanese primary teachers’ planning and classroom practices in teaching</td>
<td>72 survey respondents + 5 classroom</td>
<td>Questionnaires + video recorded lessons</td>
<td>Frequency analysis</td>
</tr>
<tr>
<td>mathematics?</td>
<td>teachers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>To what extent are mathematics teaching practices aligned with curriculum intentions?</td>
<td>5 classroom teachers + 72 survey</td>
<td>28 video recorded lessons</td>
<td>Frequency analysis</td>
</tr>
<tr>
<td></td>
<td>respondents</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What influences implementation of curriculum intentions?</td>
<td>5 Classroom teachers</td>
<td>14 audio recorded informal interviews</td>
<td>Frequency analysis</td>
</tr>
</tbody>
</table>

Chapter 4: Research Design
Given the design of the study, Chapter 5 will report the results from Phase 1 and Chapters 6 and 7 report results from Phase 2.
Chapter 5: Phase 1 Macro-level Results

This chapter presents findings from the macro-level phase of the case study design, the survey. The response rate to this survey was about 67%. As described in Chapter 4, the survey was comprised of six categories. The responses have been analysed to provide a broad picture of the practices and beliefs of Bhutanese primary school mathematics teachers. That is, the macro-level study provides an initial response to the study’s first three research questions:

Question 1: What are the beliefs of Bhutanese primary teachers about mathematics teaching?

Question 2: What are Bhutanese primary teachers’ planning and classroom practices in teaching mathematics?

Question 3: To what extent are mathematics teaching practices aligned with the curriculum intentions?

In brief, the six survey categories were: demographic data of the respondents, including gender, age level, teaching qualifications, teaching experience and geographic location; beliefs in relation to nature of mathematics; beliefs about mathematics teaching; beliefs about mathematics learning; beliefs about and acceptance of the new curriculum; and designing of sample learning activities intended for primary school mathematics classrooms. In the following sub-sections, the responses are presented with respect to these six categories of the survey. The findings are then discussed to present a broad picture of the teaching and learning of mathematics in Bhutanese primary school classrooms. This discussion leads to the identification of themes that were further explored and elaborated upon in the micro-level phase of the study presented in Chapters 6 and 7.

5.1 RESPONDENTS’ DEMOGRAPHIC INFORMATION

The demographic data section of the survey comprised eight questions, divided into two sub-sections:

- Information about the teacher (i.e., gender, qualification, age level and teaching experiences);
- Information about the teaching environment (i.e., location, job title, class level, class size).

The analysis of the demographic data first considered the two sub-sections and established that the sample of respondents was representative of the population of Bhutanese primary school teachers teaching mathematics. Then, cross-tab analysis was conducted to check for relationships between the demographic-based sub-groups of respondents.

5.1.1 **Teacher demographics**

The demographic data for the teachers who responded to the survey are discussed in this sub-section. Table 5.1 presents the gender of survey respondents. Similarly, Table 5.2, presents the qualification of survey respondents.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Frequency</th>
<th>Percentage of Total Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>50</td>
<td>62.5</td>
</tr>
<tr>
<td>Female</td>
<td>30</td>
<td>37.5</td>
</tr>
</tbody>
</table>

As indicated in Table 5.1, the sample of survey respondents is not far different from the general population of teachers in primary schools presented in Table 5.5. For instance, the percentage of males found from the survey is 63%, which is very close to the general proportion of male teachers’ in the population (i.e., 65%). Hence, the comparability of the sample and the population is the basis for asserting that in this macro-level phase of the study, any claims made regarding the beliefs and practices of the sample can be cautiously generalised to that of the whole population of primary teachers, in terms of gender.
Table 5.2
Qualifications of Survey Respondents

<table>
<thead>
<tr>
<th>Qualification</th>
<th>Frequency</th>
<th>Percentage of total respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Teaching Certificate (PTC)</td>
<td>24</td>
<td>30.0</td>
</tr>
<tr>
<td>Bachelor of Education (Primary)</td>
<td>22</td>
<td>27.5</td>
</tr>
<tr>
<td>Bachelor of Education (Secondary)</td>
<td>15</td>
<td>18.8</td>
</tr>
<tr>
<td>Other</td>
<td>15</td>
<td>18.8</td>
</tr>
<tr>
<td>No response</td>
<td>4</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Around 46% of the respondents had obtained a bachelor degree qualification in Education, either for primary or secondary mathematics education, which is slightly more than teachers with the Primary Teacher Certificate (35%). Moreover, the percentage of respondents with the Primary Teacher Certificate is very close to the general percentage of PTC teachers (36%) indicated in Table 5.6. The survey also showed that it is common for secondary-trained teachers to teach mathematics in primary schools.

Further, in Table 5.2, the category of other includes respondents with qualifications other than the Primary Teacher Certificate or Bachelor of Education in mathematics (Primary or Secondary). As discussed in Chapter 2 (Section 2.3.4), the category other is assumed to represent Bachelor of Education graduates with a non-mathematics teaching subject. As described in Chapter 2, there were several similar courses for other subjects such as English, Dzongkha, Science, Geography and History that were offered in parallel to the Bachelor of Education in Primary Maths. Irrespective of their teaching subject, all Bachelor of Education in Primary graduates were prepared to teach primary general subjects (including maths) from Classes Pre-Primary to 3. As such, there is a high chance that 15% of other respondents belong to this category of teachers.
Considering that the Primary Teacher Certificate course was phased out in 2002, 53% of the respondents had exclusively a Bachelor Degree qualification. The remaining 47% had either Bachelor Degree or Primary Teacher Certificate.

Table 5.4
Experience of the Survey Respondents

<table>
<thead>
<tr>
<th>Years of experience</th>
<th>Frequency</th>
<th>Percentage of total respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>24</td>
<td>30</td>
</tr>
<tr>
<td>4-7</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>8-19</td>
<td>30</td>
<td>38</td>
</tr>
<tr>
<td>&gt;19</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>No response</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

As indicated in Table 5.4, slightly more than half (55%) of respondents were in their first seven years of teaching. In the history of the Bhutanese mathematics education, the concept of NCTM standards was integrated into the pre-service programs (B. Ed in Mathematics) from 2003, prior to the development of the new curriculum. Assuming these respondents moved straight from their teaching training into the workforce, they would have had some exposure to the NCTM based principles of the new primary curriculum. Tables 5.5 and 5.6 present brief information regarding the teaching population of primary and secondary qualification of teachers in the country (Ministry of Education, 2013b).
Comparison of the survey respondents and its population show that the sample is similar in terms of gender and qualifications composition, to the general population of Bhutanese teachers.

5.1.2 Teaching environment demographics

The second demographic sub-section of the survey explored the teaching environment of the respondents. The responses in relation to geographical location are summarised in Table 5.7.

Although Bhutan is a small country, the concept of urban and rural can be easily defined. If the school is located within ten kilometres from a town, it is referred to as urban and if it is between 10 and 18 km from a town, it is semi-urban. A school
located more than 18 kilometres from a town can be classed as rural. If the school is located where it takes a day or two to travel there on foot, from the nearest town, then it can be termed remote. As indicated in Table 5.7, the respondents were spread across all geographic locations with only 28% representing urban; 45% were represented by rural and remote, and 26% under semi-urban.

Table 5.7

<table>
<thead>
<tr>
<th>Position</th>
<th>Count</th>
<th>Percentage of total respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classroom Teacher</td>
<td>74</td>
<td>93</td>
</tr>
<tr>
<td>Head of Department</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Vice-Principal</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Principal</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>No Response</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.8 reveals that a few of respondents (9%) had other responsibilities in addition to their normal teaching loads.

Table 5.8

Position of the Survey Respondents

Table 5.9

<table>
<thead>
<tr>
<th>Class</th>
<th>Count</th>
<th>Percentage of Total Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class PP-3</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>Class 4</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>Class 5</td>
<td>25</td>
<td>31</td>
</tr>
<tr>
<td>Class 6</td>
<td>17</td>
<td>21</td>
</tr>
<tr>
<td>No Response</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

In terms of the Class level taught, 25% of respondents were involved in teaching lower primary (Class Pre Primary – 3) and 72% taught upper primary classes (Class 4 – 6).
In terms of class size, approximately 38% of respondents had smaller classes (30 and below) and the remainder, 61%, had larger classes (40 and above). This finding is in contrast with the target set by the Queensland Education Department, which is 25 students per class (Vonow, 2015).

In conclusion, when considered together, the data in this sub-section suggests that most of the respondents were classroom teachers with no other responsibilities. The majority taught students in upper-primary classes with the number of students in other classes ranging from 31 to 40. Having presented the results from respondents’ demographic data separately, findings are explored further in terms of cross-tab analysis presented in the following sections.

5.1.3 Cross-tab analysis

Although representativeness has been established across the entire sample in terms of gender and qualifications, it is also important to consider the sub-groups of teachers (based upon the various demographic differences) and consider how these differences might be related. For example, a teacher in a rural setting could face more challenges in terms of gaining access to learning and teaching resources compared to those in urban settings. Hence, to help identify such influences, a series of cross-tab analyses were done within demographic features (i.e. location, qualification, teaching experience and class size). The following potential relationships were explored:

- Teacher qualification and teaching experience;
- Teacher qualification and geographical location; and
- Geographical location and class size.


**Teacher qualification and teaching experience**

Teachers’ qualifications and teaching experience can both have an impact on the learning environment. According to Darling-Hammond and Post (2000), student achievement depends on the teacher’s qualification and experience. Darling et al. (2010) found that teacher qualifications in particular were important to students’ achievement. In Bhutan, a teacher with an advanced qualification, such as a degree, is always considered to be a great boon to the respective school in terms of content knowledge and student achievement. In Table 5.11, the cross-tab comparison of the respondents’ qualifications and years of experience is presented.

Table 5.11

*Cross-Tab Analysis (Qualification and Teaching Experience)*

<table>
<thead>
<tr>
<th>Qualification</th>
<th>Teachers’ Years of Teaching Experience</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1-3 Years</td>
</tr>
<tr>
<td>--------------------------------------</td>
<td>------------</td>
</tr>
<tr>
<td>Primary Teacher Certificate (PTC)</td>
<td>2</td>
</tr>
<tr>
<td>Bachelor of Education in Primary Mathematics</td>
<td>6</td>
</tr>
<tr>
<td>Bachelor of Education in Secondary Mathematics</td>
<td>7</td>
</tr>
<tr>
<td>Other qualification</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 5.11 shows that in Bhutanese primary school mathematics classrooms, the more experienced teachers tend to be those with the PTC qualification. Conversely, teachers with other qualifications, including a bachelor’s degree, tend to have fewer years of experience. This pattern was expected, since the PTC course was phased out after 2002. Moreover, during the time of data collection, some of the teachers were expected to be recently graduated with a four-year degree (Bachelor of Education in Primary Curriculum), introduced in 2009. Thus, the more recently graduated teachers with less experience have a higher-status qualification, and they have been exposed to the principles underpinning the new curriculum. Teachers with more experience were typically less qualified and, as a consequence, had not been exposed to the underpinning principles of constructivism as part of their pre-service training.
Teachers’ qualification and geographic location

From one perspective, the qualification of the teacher should not matter in terms of location of the school. As long as the teacher is confident with teaching materials, skills and strategies could be easily adjusted no matter the location. However, understanding the geographic distribution of teachers within the sample is important because in Bhutan, inexperienced graduate teachers are sent to rural or remote schools for a minimum period of three years to replace senior teachers who move back to urban schools. Similarly, researchers such as Darling-Hammond and Post (2000) have pointed out that US teachers who are less qualified are often appointed to schools in rural or low socio-economic areas. Table 5.12 represents a cross-tab analysis of the respondents’ qualification and geographic location.

Table 5.12
Cross-Tab Analysis (Qualifications and Geographical Location)

<table>
<thead>
<tr>
<th>Teacher’s Qualification</th>
<th>Geographic Location</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Urban</td>
</tr>
<tr>
<td>Primary Teacher Certificate</td>
<td>11</td>
</tr>
<tr>
<td>Bachelor of Education (Primary)</td>
<td>5</td>
</tr>
<tr>
<td>Bachelor of Education (Secondary)</td>
<td>5</td>
</tr>
<tr>
<td>Other qualification</td>
<td>1</td>
</tr>
</tbody>
</table>

As shown in Table 5.12, 44 of the respondents were from urban or semi-urban schools. Of these, 21 had a PTC qualification. The majority of the respondents with PTC qualifications were therefore, in semi-urban or urban schools. They were likely to be experienced teachers, who had been transferred from rural to urban schools. Around 43% were from remote schools. Of these, 35% had a Bachelor of Education in primary qualification. Therefore, the predicted pattern based on employment policies is evident in this sample. Similarly, in the next section, the location of the school is further explored related to the number of students in the classroom.

Geographic location and class size

Bhutan is a mountainous country and so geography plays an important role in terms of development of the country and the resourcing of schools. This relationship was explored to see if the location of a school and class size were related. Such a
relationship was found to be very important in terms of helping students with quality learning, as stated by Darling-Hammond and Post (2000): “teacher education, ability, and experience, along with small schools are associated with significant increases in student achievements” (p. 131). However, in the Bhutanese context, schools located in far flung (remote) areas are typically poorly equipped with teaching and learning materials, mainly due to the difficulties faced in transporting materials such as textbooks, teaching resources and stationery. As indicated in Figure 5.1, unlike in developed countries like Australia, the difference between urban and rural is very distinct. Compared to rural and remote areas, urban areas are thickly shaded to show densely populated.

![Figure 5.1. A map of the population density of Bhutan.](image)

In Table 5.13, the cross-tab analysis of geographic location and class size is presented.

<table>
<thead>
<tr>
<th>Class size</th>
<th>Urban</th>
<th>Geographic Location of School</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Urban</td>
</tr>
<tr>
<td>&lt;21</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>21-30</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>31-40</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>&gt;40</td>
<td>10</td>
<td>2</td>
</tr>
</tbody>
</table>

As expected, the survey data analysis shows a pattern of larger classes in urban and
semi-urban schools, and smaller classes in rural and remote schools. This reflects the current rural to urban migration pattern in the country.

5.1.4 **Summary**

The findings from the 80 respondents suggest that the distribution of both gender and qualifications was closely representative of the overall primary teacher population in the country. Further, the results tend to reveal that teachers with the most years of teaching experience had Primary Teacher Certificate qualifications compared to Bachelor of Education in Primary Mathematics, with the longest service of 19 years. Moreover, most of the teachers serving in semi-urban to urban areas were also Primary Teacher Certificate graduates. The following section explores respondents’ beliefs about mathematics and mathematics education.

5.2 **TEACHERS’ BELIEFS ABOUT MATHEMATICS AND MATHEMATICS EDUCATION**

In this section, responses to 21 belief items measured using a five-point Likert scale are presented. Descriptive statistics were calculated for each question. Although the questions were derived from the survey developed by Perry et al. (1999), due to the small size sample (n=80), attempts to replicate the factor structure were unsuccessful. Hence, responses to each category of belief statements are reported for each item in the following sections.

5.2.1 **The nature of mathematics**

Beliefs statements about the nature of mathematics were one of the sub-sections of the survey. As discussed in Section 3.1.1, beliefs are related to personal understanding grounded on ideas about the world (Charalambous et al., 2009). Similarly, respondents’ beliefs about the nature of mathematics were explored through their responses to six items. The items and descriptive statistics are presented in Table 5.14. A higher score indicates stronger agreement.
The majority of respondents indicated strong agreement for Items 1, 3 and 4. Compared to Items 3 and 4, the result for Item 1 appeared to support instrumental beliefs about the nature of mathematics. The mean score of 4 indicates that the majority of the respondents either agreed or strongly agreed with Item 1 (mathematics is computation). This may indicate a narrow perspective about the nature of mathematics in terms of alignment with the intentions of the new curriculum. However, there is a possibility that the respondents did not understand the statement and interpreted it as mathematics includes computation, including those teachers involved in piloting the survey. Having agreed with Items 3 and 4, the majority of respondents tended to indicate experimental beliefs about the nature of mathematics, as defined by Ernest (1989), discussed in Chapter 3. Although Item 4 is a combination of three concepts (beautiful, creative and useful), all are assumed to be in support of ideas associated with the constructivist approach. Moreover, by disagreeing or strongly disagreeing with Items 5 and 6, the majority of respondents further expressed their beliefs aligned with the curriculum intentions.

The general disagreement with Items 5 and 6 suggests that most respondents in this sample did not believe that mathematics is knowledge isolated from other knowledge, and that they placed more importance on process than answers. Given a
mean and median of 3 for Item 2, respondents neither agree nor disagree with the proposition. On balance, the overall result suggest that the majority of respondents favour experimentalist beliefs about the nature of mathematics, thus aligning with the curriculum intentions, discussed in Chapter 2. This is, in turn, expected to shape their beliefs about mathematics teaching and learning, which are explored in the following sections.

5.2.2 Respondents’ beliefs about mathematics learning

The way teachers conduct mathematics lessons could reflect the type of beliefs they have about how students should learn mathematics, as discussed in Chapter 3. For instance, if the teacher’s beliefs about mathematics and mathematics learning align with experimentalist beliefs (Section 3.1.1), students would be expected to be engaged in learning activities in which they construct knowledge for themselves. As a consequence, students’ understanding level is likely to be relational (Skemp, 1976). On the other hand, a teacher with instrumental beliefs about mathematics might be expected to conduct lessons in which students receive knowledge passively and are expected to learn through memorisation of formulas and rules to be applied directly without much understanding. Consequently, students end up with an instrumental understanding (Skemp, 1976), and gain procedural knowledge of mathematics. The respondents’ responses to the items concerning beliefs about the learning of mathematics are presented in Table 5.15.
Table 5.15  
*Distribution of Responses for Mathematics Learning*

<table>
<thead>
<tr>
<th>Item</th>
<th>Median</th>
<th>IQR</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. Mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences.</td>
<td>4.0</td>
<td>0.0</td>
<td>3.9</td>
<td>0.66</td>
</tr>
<tr>
<td>8. Children are rational decision makers capable of determining for themselves what is right and wrong.</td>
<td>4.0</td>
<td>1.0</td>
<td>3.90</td>
<td>0.90</td>
</tr>
<tr>
<td>9. Mathematics learning is being able to get the right answers quickly.</td>
<td>3.0</td>
<td>2.0</td>
<td>2.82</td>
<td>0.98</td>
</tr>
<tr>
<td>10. Periods of uncertainty, conflict, confusion, and surprise are a significant part of the mathematics learning process.</td>
<td>4.0</td>
<td>1.0</td>
<td>3.81</td>
<td>0.85</td>
</tr>
<tr>
<td>11. Young children are capable of much higher levels of mathematical thought than has been suggested traditionally.</td>
<td>4.0</td>
<td>1.0</td>
<td>3.45</td>
<td>0.91</td>
</tr>
<tr>
<td>12. Mathematics learning is enhanced by activities which build upon and respect students’ experiences.</td>
<td>4.0</td>
<td>0.0</td>
<td>4.17</td>
<td>0.52</td>
</tr>
<tr>
<td>13. Children construct their own mathematical knowledge.</td>
<td>4.0</td>
<td>0.0</td>
<td>3.76</td>
<td>0.72</td>
</tr>
</tbody>
</table>

The responses to Items 7, 10, 12 and 13 suggest that most respondents believed that it is very important for teachers to provide students with meaningful and context based learning activities. Providing such a learning environment can encourage students to connect their prior knowledge both vertically and horizontally to make sense in mathematics. The responses given to Item 13 suggest that respondents at least have a notion that the students are capable of constructing their own knowledge in favourable conditions. However, the statement did not explicitly describe such conditions. Hence, the responses to Items 7, 10, 12 and 13 indicate that the majority of respondents appeared to have beliefs about mathematics learning which were aligned with the intentions of the new curriculum.

Further, the responses to Item 11 suggest that teachers believed young children are capable of higher levels of mathematical thought than has been suggested traditionally. For Item 9, the median is 3 suggesting that they were fairly evenly divided in terms of the importance of right answers. Hence, the data suggest that there was a sign of awareness in the respondents regarding the importance of
encouraging children in their mathematical thinking processes and that they are capable of higher levels of thought.

The majority of respondents indicated beliefs about mathematics learning that were aligned with the intention of the new curriculum, particularly by their responses to Items 7, 8, 12 and 13. In all of these items, both mean and median scores were above 3, indicating a strong overall support for the statements. The content of these items was about students being able to construct their own knowledge with the provision of favourable learning situations aligning with curriculum (i.e. use of context). Hence, these results indicate respondents’ beliefs about mathematics education, supporting experimental beliefs and aligned quite strongly with the four curriculum intentions. These findings tend to indicate that the majority of respondents were aware of how students are expected to learn mathematics. Teachers’ beliefs about the teaching of mathematics are presented in the next section.

5.2.3 **Respondents’ beliefs about mathematics teaching**

According to the literature on beliefs about mathematics and mathematics teaching, as previously discussed in Chapter 3, teachers’ practice in the classroom depends on the type of beliefs they hold. For example, a teacher with experimental beliefs could be expected to conduct lessons that encourage students to construct their own knowledge. Respondents’ beliefs about how students should be taught mathematics were explored through their responses to eight items. The items and descriptive statistics are presented in Table 5.16.
Table 5.16
Distribution of Responses for Mathematics Teaching

<table>
<thead>
<tr>
<th>Item</th>
<th>Median</th>
<th>IQR</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>14. Teachers should provide instructional activities which result in problematic situations for learners.</td>
<td>4.0</td>
<td>1.0</td>
<td>3.54</td>
<td>1.11</td>
</tr>
<tr>
<td>15. Teachers or the textbook – not the student – are the authorities for what is right or wrong.</td>
<td>2.0</td>
<td>1.0</td>
<td>2.38</td>
<td>1.12</td>
</tr>
<tr>
<td>16. The role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge.</td>
<td>4.0</td>
<td>1.0</td>
<td>3.67</td>
<td>1.11</td>
</tr>
<tr>
<td>17. Teachers should recognise that what seems like errors and confusions from an adult point of view are children’s expressions of their current understanding.</td>
<td>4.0</td>
<td>1.0</td>
<td>3.64</td>
<td>0.87</td>
</tr>
<tr>
<td>18. Children’s development of mathematical ideas should provide the basis for sequencing topics for instructions.</td>
<td>4.0</td>
<td>0.0</td>
<td>4.00</td>
<td>0.65</td>
</tr>
<tr>
<td>19. It is unnecessary, even damaging, for teachers to tell students if their answers are correct or incorrect.</td>
<td>2.0</td>
<td>1.0</td>
<td>2.62</td>
<td>1.07</td>
</tr>
<tr>
<td>20. Mathematics skills should be taught in relation to understanding and problem solving.</td>
<td>4.0</td>
<td>1.0</td>
<td>4.46</td>
<td>0.60</td>
</tr>
<tr>
<td>21. Mathematics instruction should be organised to facilitate children’s construction of knowledge.</td>
<td>4.0</td>
<td>1.0</td>
<td>4.38</td>
<td>0.65</td>
</tr>
</tbody>
</table>

The majority of respondents either agreed or strongly agreed with Items 14, 16, 17, 18, 20 and 21. On the other hand, the majority of respondents either disagreed or strongly disagreed with Items 15 and 19. These responses suggest that the majority of the respondents had constructivist (i.e., experimentalist) beliefs about mathematics teaching. The majority of the respondents supported Item 21 and believed that it was important for teachers to design instruction that is well-organised and facilitates students’ construction of knowledge. A vertical connection is related to children’s knowledge within mathematical topics, and the horizontal connection is related to other knowledge experienced outside the classroom (Treffers, 1987). The responses suggest the importance of relating the concept taught to children’s own experiences in terms of its connection both horizontally and vertically.

Further, the analysis for Items 15 and 19 are interpretable as supporting instrumental beliefs to teaching mathematics. The majority of respondents either dis-
agreed or strongly disagreed to Items 15 and 19, not supporting the statements. This is consistent with their support for experimental beliefs that align with curriculum intentions.

### 5.2.4 Summary

The overall results tend to suggest that the majority of respondents expressed experimentalist beliefs about mathematics supporting a constructivist approach to the teaching and learning of mathematics, thereby aligning them with the intention of new curriculum. For instance, the majority of respondents supported the notion that children are rational decision-makers and capable of determining what is right and wrong. They were also of the opinion that children are capable of constructing their own mathematical knowledge based on the provision of favourable learning conditions and support from the teacher. Students require supportive scaffolding for the construction of knowledge. It is less likely they will construct their own knowledge without guidance. Having explored respondents’ beliefs about mathematics and mathematics education in the previous sections, the following section describes respondents’ beliefs about the new curriculum.

### 5.3 NEW MATHEMATICS CURRICULUM

As discussed in Chapter 2, the new curriculum was based upon constructivist principles that were different from the underpinnings of the previous curriculum, which had been framed around a traditional teacher-centred chalk and talk method of teaching. In the fifth category of the survey, there were six items describing beliefs about and acceptance of the new curriculum. These items were developed specifically for this study. The reason for doing this was to explore the relationship between the responses of the respondents’ beliefs about mathematics and mathematics education, and how these aligned with their beliefs and acceptance of the new curriculum. These six items, and their descriptive statistics, are presented in Table 5.17, which is followed by a discussion of the responses.
Table 5.17

Respondents’ Beliefs about the New Curriculum

<table>
<thead>
<tr>
<th>Item</th>
<th>Median</th>
<th>IQR</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>22. The new mathematics curriculum is realistic and relevant</td>
<td>4.0</td>
<td>1.0</td>
<td>4.14</td>
<td>0.79</td>
</tr>
<tr>
<td>23. The new mathematics curriculum is challenging but interesting for students</td>
<td>4.0</td>
<td>1.0</td>
<td>4.15</td>
<td>0.79</td>
</tr>
<tr>
<td>24. The new mathematics curriculum is well connected within and beyond the curriculum</td>
<td>4.0</td>
<td>0.75</td>
<td>4.09</td>
<td>0.65</td>
</tr>
<tr>
<td>25. The new mathematics curriculum is complicated and difficult to follow</td>
<td>2.0</td>
<td>1.0</td>
<td>2.59</td>
<td>0.99</td>
</tr>
<tr>
<td>26. The guidebook provided is very helpful and enriching</td>
<td>4.0</td>
<td>1.0</td>
<td>4.14</td>
<td>0.73</td>
</tr>
<tr>
<td>27. The idea of providing learning at the beginning of every lesson in the textbooks activities (e.g. the TRY THIS section) is a very enriching idea for the student</td>
<td>4.0</td>
<td>0.0</td>
<td>3.83</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Responses to Items 22, 23, 24, 26 and 27 suggest that the majority of respondents had a positive disposition and accept the new curriculum as realistic, interesting, challenging, and well-connected within and beyond the curriculum. In particular, responses to Item 26 indicate that respondents believed that the Teachers’ Guidebook was very helpful and informative. Similarly, the idea of beginning the lesson with a thought-provoking activity (Item 27) was accepted as very useful and enriching for students. Item 25, which was a negatively-phrased statement that the new curriculum is complicated and difficult to follow, was widely disagreed with. Together, these responses provide evidence that the respondents were aware of what was intended by the new curriculum and were, in general, in favour of its intentions.

Having revealed respondents’ pre-dominantly positive disposition towards the intention of the new curriculum, the results from this section tends to support claims made regarding the respondents’ experimental beliefs about mathematics and mathematics teaching, thus addressing Research Question 1: What are the beliefs of Bhutanese primary teachers about mathematics teaching? Further, the results of this analysis align with the positive feedback received from practising teachers in Bhutan during presentation of the new curriculum to various schools and teachers (Curriculum and Professional Support Division, 2005).
However, the new curriculum’s greatest innovation was based upon its philosophical intentions in terms of the conceptual delivery of mathematical topics. The purpose of the current study is intended to explore the actual implementation of those curriculum intentions. With this in mind, a request to suggest a sample learning activity was included in the survey to explore the actual practices of respondents in terms of alignment with planning and implementation of the new curriculum, as presented in the following section.

5.4 SAMPLE LEARNING ACTIVITIES

As discussed in the previous sections, the majority of respondents seemed to hold beliefs supportive of the new curriculum. According to the Bhutanese curriculum framework, a strong emphasis was placed on students generating their own ways of solving problems through a relational understanding of mathematics rather than memorising a superficial assortment of rules and definitions (Ministry of Education, 2003). In order to enact such practices in the classroom, teachers were expected to design learning activities that enabled students to learn mathematics meaningfully (NCTM, 2000). Hence, information from the sixth category of the survey explored how teachers might enact their beliefs.

This section analyses the sixth category of the survey in which respondents’ designed a learning activity. Respondents were provided with a list of lesson objectives related to the teaching of fractions from Pre-Primary to Grade 6 (refer to Appendix B). Respondents were asked to choose one objective from the list and design a sample learning activity for a class. The intention of this section of the survey was to identify how teachers would plan to implement an element of the new curriculum in their classrooms. Based upon findings of past studies, teachers’ planning depends on their beliefs about mathematics and mathematics education. Hence, the design of the learning activity was expected to expose teachers’ beliefs about mathematics and mathematics education.

Of the 80 survey respondents, 72 attempted to design a learning activity as requested. To enhance validity and reliability of the analysis, two approaches were used to analyse the data, and determine the sample activities’ alignment with the four curriculum intentions. This analysis is presented in four sub-sections, corresponding
to each of the four intentions of the curriculum. This is followed by an overall
discussion of the beliefs evident from the sample learning activities.

5.4.1 **Intention 4: Use of process standards**

This section presents the analysis of the 72 sample learning activities in regards to
the opportunities they provided for students to apply the five process standards. Each
of the sample learning activities was mapped against a process standards framework
adapted from NCTM (2000) as presented in Table 5.18. In turn, this was expected to
reveal aspects of the respondents’ beliefs about mathematics in terms of students’
engagement in learning.

Prior to using the process standards framework for the sorting of sample
learning activities, a pilot test was conducted with a group of lecturers/teachers with
a mathematics background, as discussed in Chapter 4, Section 4.6.1. Using the same
coding strategy, the 72 samples were analysed, the results of which are presented in
Table 5.18. In that table, the frequency of occurrence (represented as a percentage)
was calculated by tallying how many sample activities included some aspect of the
standard, divided by the total number of samples (72). The quality or complexity of
each standard’s integration into the sample learning activity was not taken into
account.
Table 5.18

*Process Standard Learning Activity Analysis*

<table>
<thead>
<tr>
<th>Process Standard</th>
<th>Indicators</th>
<th>Frequency of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem solving</td>
<td>Solve problems more meaningfully through the act of applying prior knowledge and methods to a task for which the solution is not apparent;</td>
<td>13%</td>
</tr>
<tr>
<td>Communication</td>
<td>Communicate their thinking and expression of mathematical ideas precisely in the form of speaking, writing and reading;</td>
<td>18%</td>
</tr>
<tr>
<td>Connections</td>
<td>Connect and enable the recognition of the relationships among mathematical topics as well as applications of mathematics in other contexts;</td>
<td>96%</td>
</tr>
<tr>
<td>Representations</td>
<td>Represent their understanding through various forms (i.e., enactively, iconically &amp; symbolically) and describe mathematical relationships;</td>
<td>65%</td>
</tr>
<tr>
<td>Reasoning and proof</td>
<td>Justify one’s mathematical thinking as well as evaluate the mathematical thinking of others.</td>
<td>17%</td>
</tr>
</tbody>
</table>

The majority of sample learning activities included opportunities for two of the process standards: connections (96%), followed by representation (65%). One of the examples indicating these process standards is shown in Figure 5.3. The least frequently evident standard was problem solving, with only 13% of the samples including challenging, non-routine tasks that would provide opportunities for multiple solution methods. Likewise, few respondents attempted to design a task that might encourage students to explain their mathematical reasoning and articulate their thinking used to solve the problem.

The analysis was further refined by recoding the samples using more specific forms of the connection, representations and communication standards to generate findings with deeper meaning. This was done to obtain in-depth information regarding those standards in terms of practice implemented by the respondents. This analysis is presented in Table 5.19. For instance, the majority of respondents integrated the concept of connection in the design of their sample learning activity but it was not clear which of the two connections (horizontal and vertical) was referred to. The same idea was applied for communication and representation, as presented in Table 5.19.
Table 5.19

*Fine-Grained Analysis of Sample Learning Activities*

<table>
<thead>
<tr>
<th>Connection</th>
<th>Representation</th>
<th>Communication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>Horizontal</td>
<td>Enactive</td>
</tr>
<tr>
<td>96%</td>
<td>35%</td>
<td>17%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Iconic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>57%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Symbolic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>57%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Verbal</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Written (Words)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Written (Numerals)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50%</td>
</tr>
</tbody>
</table>

The sub-divisions of the process standards indicated in Table 5.19 were derived from learning theorists related to the constructivists’ approach of teaching and learning of mathematics. For instance, according to Treffers (1987), connection can be divided into two: connecting vertically (within mathematical ideas) and horizontally (connecting mathematical knowledge across other bodies of knowledge/real world). Similarly, according to Bruner (1966), learners are expected to be able to express their understanding of mathematical understanding in three different modes: enactive (expression of mathematical understanding in action), iconic (expression of mathematical understanding in pictures/diagrams) and symbolic (expression of mathematical understanding in words/numerals/symbols). Similarly, the National Council of Teachers of Mathematics (2000) stresses encouraging students to communicate their thinking through three different ways, communicating with another either verbally or in written words or symbols.

As indicated in Table 5.19, the vast majority (96%) of the respondents incorporated an opportunity for constructing vertical connections; however, horizontal connections were less frequent, appearing in only 35% of the samples. This is despite the curriculum suggesting that both kinds of connections should be treated with equal importance. An example sample learning activity which incorporated both horizontal and vertical connections was:

Sonam, Tashi and Ugyen were given one litre of water (1 litre) each. Sonam drank 2/6 of the water; Tashi drank 4/8 of the water and Ugyen drank 1/9 of the water. Draw models to answer the questions (*who drank more water?*).

In this sample activity, vertical connections are incorporated when measuring (e.g., concept of measurement and capacity) the quantity of water in fractions, and horizontal connections are incorporated by the use of children’s experiences of drinking any kind of liquid.
In terms of the representation standards, the sample activities tended to include more iconic and symbolic representations than enactive, both scoring 57%. This contrasts to the use of enactive representations, which only appeared in 17% of the sample activities. An example of an activity which encouraged students to represent their thinking and understanding through the conduct of action (i.e., enactively) is reproduced in Figure 5.2.

![Sample learning Activity A](image)

**Figure 5.2. Sample learning Activity A.**

The first statement in Figure 5.2 provides an opportunity for students to represent their thinking and understanding of the assigned task in action (enactively). “*Have the student cut out 2D shapes (e.g., circle) and let them divide the shape equally into two parts*. In this sample, the enactive mode of representation was clearly encouraged by asking students to act out the process of cutting and dividing the given circle. According to Bruner (1966), learners are said to have mastered a concept if they can represent their understanding through these three (enactive, iconic and symbolic) modes of representations simultaneously. Hence, in this activity students could be described as using the reasoning indicator from the 3+1 analytical framework, *flexibly use of written and spoken language as well as symbolic, iconic and enactive representations*. However, learning activities of this kind were infrequent among the 72 samples analysed. The majority of the sample learning activities appeared to be more like lesson plans, as reproduced in Figure 5.3.
Figure 5.3 Sample learning Activity B.

In sample learning Activity B, it appears that the respondent intended to explain the strategy of comparing fractions using the same denominator with a statement, “if the fractions have the same denominator, the fraction with the greater numerator is greater, e.g. $\frac{4}{7} \lt \frac{6}{7}$ because 6 sevenths > 4 sevenths (6 of something is more than 4 of the same thing).

In this case, there is hardly any opportunity given to students to explore and construct their own meaning rather than listening to the teacher’s explanation. There was no indication in the task of students performing anything themselves. Instead, the sample appeared to be an information input from the teacher rather than a learning task to be given to students by which they might construct their own knowledge using the process standards. As argued by Stein and Lane (2006), such teacher-centred transmission limits the opportunity for students to do mathematics through manifold approaches with multiple representations and explanations. Students were also deprived of opportunities for exploring and understanding the concept.

Of all the process standards, communication was evident in only 18% of sample activities and these were mostly focussed on writing numerals rather than talking, discussion or written narratives. The majority of respondents avoided incorporating the process standards reasoning, communication and problem solving. This result suggests that the dominant practice of conducting mathematics lessons is
still the talk and chalk method, where students are encouraged to be passive listeners. Such approaches are likely to lead to surface learning and procedural knowledge (Skemp, 1976).

Pursuing this further, the researcher determined to explore the findings in terms of respondents’ qualifications. This was done mainly to compare the practices of teachers with and without exposure to some of the learning theories aligning with the curriculum intentions. Table 5.20 presents a similar analysis to that in Table 5.19 except the frequency analysis was split between the PTC (n = 26) and Bachelor of Education in Primary mathematics (n = 31) respondents. It should be noted that the 15 respondents with other qualifications were not included in this comparative analysis.
Table 5.20

Comparison of Standards within the Two Qualifications

<table>
<thead>
<tr>
<th>Qualifications</th>
<th>Connections</th>
<th>Use of Process standards</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vertical</td>
<td>Horizontal</td>
<td>Enactive</td>
<td>Iconic</td>
<td>Symbolic</td>
<td>Verbal</td>
</tr>
<tr>
<td>PTC (n=26)</td>
<td>24</td>
<td>10</td>
<td>5</td>
<td>11</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>93%</td>
<td>39%</td>
<td>54%</td>
<td>49%</td>
<td>50%</td>
<td>54%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>11%</td>
<td>49%</td>
<td>11%</td>
<td>49%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>54%</td>
<td>12%</td>
<td>11%</td>
<td>3%</td>
</tr>
<tr>
<td>B.Ed Pry Maths (n = 31)</td>
<td>29</td>
<td>11</td>
<td>4</td>
<td>22</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>93%</td>
<td>50%</td>
<td>18%</td>
<td>71%</td>
<td>65%</td>
<td>13%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3%</td>
<td>56%</td>
<td>3%</td>
<td>3%</td>
</tr>
</tbody>
</table>

Note: For each qualification, the occurrence of each feature is expressed both as a frequency count and percentage of corresponding qualification.
Compared to Bachelor of Education graduates, respondents with Primary Teacher Certificate (PTC) opted slightly more for process standards such as *enactive representation* (54%); *verbal* (19%) and *written word communication* (11%); and *reasoning* (12%). PTC teachers seem to have shown better plans for fostering enactive representation, verbal communication and reasoning compared to the Bachelor of Education graduates. These teachers are able to encourage students to communicate their thinking and ideas enactively with adequate justification, which are considered as key elements in the development of deep understanding of the targeted concepts. Similarly, a deep understanding in mathematics is evident when children are able to express both orally and in written words, as well as writing numerically (National Council of Teachers of Mathematics, 2000) and also enactively (Bruner, 1966). Having incorporated these stated standards, PTCs performed better than Bachelor of Education graduates in terms of their alignment with the child-centred approach, indicating implementation of the ideas intended in the new curriculum.

On the other hand, respondents with Bachelor of Education in Primary Mathematics scored higher than PTCs in iconic representation (71%) and symbolic communication (65%); and 50% for connecting horizontally. Encouraging students to represent mathematics symbolically and communicating numerically could possibly lead them to more abstract and less meaningful learning, particularly at the level of foundation building (primary schooling). In this way, students might develop surface understanding, leading towards procedural knowledge.

Respondents with Bachelor of Education qualifications appeared more likely to design learning with transmission approaches to teaching and learning of mathematics. Compared to teachers with PTC, Bachelor of Education graduates were expected to have expressed a more constructivist approach to teaching mathematics, because they had been exposed to theories of teaching and learning during their training period. Similarly, the researcher had expected that PTC graduates could reveal stronger support for instrumental beliefs since they had been trained without learning theories content. However, Bachelor of Education graduates demonstrated a stronger leaning toward practices supporting Platonist beliefs.

Pursuing this idea further, three of the standards (connections, representation and communication) were analysed in-depth in terms of their sub-components,
revealing that vertical connection, symbolic representation and communication in written numerals were the most prominent. This result indicates less connection horizontally or encouraging students to act out their understanding enactively. However, with regards to the way the majority of respondents manifested their practice through designing a sample learning activity, their beliefs tend to fall under either instrumental or Platonist beliefs.

In the following sections, analysis of the sample learning activities is conducted with regards to the first three intentions of the curriculum using the 3+1 framework. This provides a more detailed account regarding the respondents’ planned practices and underlying beliefs regarding mathematics and mathematics teaching.

5.4.2 Intention 1: Emphasis on understanding

An emphasis on understanding is about helping students to acquire deep and meaningful knowledge of mathematical concepts. In Chapter 3, the 3+1 framework was proposed to analyse classroom practices in terms of alignment with curriculum intentions. In essence, the framework provides indicators of desirable teacher and student practices in terms of their alignment with the first three intentions of the curriculum (the fourth intention being embedded in the student-practice descriptors for each of the first three intentions). Using the 3+1 rubric for understanding the complete set of sample learning activities was coded and presented in Table 5.21.

Each sample learning activity was coded against the indicators and counter indicators of teacher and student practice. In the following paragraphs, samples of learning activities are presented to illustrate the various indicators and counter-indicators of the framework. Then, comments are made regarding the prevalence of elements of the activities related to emphasising the development of understanding.
Table 5.21

*Frequency Analysis on Intention One (Relational Understanding)*

<table>
<thead>
<tr>
<th>Indicators</th>
<th>What the Teacher Does</th>
<th>%</th>
<th>What Students Do</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Use learning activities that explore and scaffold construction of conceptual schema:</td>
<td></td>
<td>Participate in activities in which they identify and describe the developing horizontal and vertical connections within their own conceptual schema</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Horizontally between students’ reality (i.e., daily-life activities or situations that can be readily imagined by the students) and mathematical activities</td>
<td>96</td>
<td></td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>- Vertically between mathematical ideas of varying levels of abstraction or sophistication</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Counter-indicators</td>
<td>Use of learning activities which do not incorporate opportunities to explicitly explore the connections between mathematical ideas</td>
<td>69</td>
<td>Passively listen to teacher explanations</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>Do not describe the vertical and horizontal connections they perceive between mathematical ideas</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results of the coding are described in terms of the frequency (expressed as a percentage) by which the indicators were evident in the samples. As with the coding of the samples using the process standards (presented in the previous section), the coding did not take into consideration the complexity or quality of each indicator’s presence in the samples. The results of the frequency analysis tend to show that across the 72 sample learning activities, there were very few opportunities for students to engage in activities that may assist in developing horizontal and vertical connections (22%). Instead, most activities could be described as teacher-led (what the teacher does) expositions (96%), albeit explaining the horizontal and vertical connections, in which students (what students do) were passive recipients of knowledge (43%). Furthermore, the way a teacher presents a problem, by explaining the solution, can lower the level of challenge (Diezmann & Watters, 2001).
The overall observation derived from the analysis of the 72 sample learning activities indicates very little involvement of students in the task. The vast majority of sample activities described what the teacher would be doing and not how students would be engaged. In these activities, students could be described using the counter-indicator passively listen to teacher explanations (43%).

An example showing such passive reception of teacher explanation is presented in Figure 5.4, which begins with “Show an apple and say this is one whole [emphasis added]”.

![Figure 5.4. Sample learning Activity C.](image)

Although this activity does attempt to make some horizontal connection to a daily-life activity, it does not require the student to actually participate in the activity, as suggested by the student-practice indicator. Many of the sample learning activities followed a similar structure in which involvement of students to construct their own knowledge was rare. Such activities proposed (or used) by the teachers do not incorporate opportunities for students to explicitly explore the connections between mathematical ideas. Nor do they encourage students to describe the vertical and horizontal connections they perceive between mathematical ideas, opportunities for students to explore and construct conceptual knowledge by connecting horizontally between their realities and mathematical ideas.

However, the example in Figure 5.5 illustrates how students are provided with opportunities to participate in activities in which they identify and describe the developing horizontal and vertical connections within their own conceptual schema. For instance, students are asked to think deeply and communicate their ideas either
verbally or in written form. At the same time, students might be encouraged to transform from the iconic form to other possible forms such as enactive (physically sharing three apples among four friends equally in action) or symbolically \((3 \div 4)\).

\[
\text{Activity to children}
1. \text{What division is being modelled in the picture?}
\]

*Figure 5.5. Sample learning Activity D.*

The preceding examples serve to illustrate the way in which the researcher interpreted the 3+1 descriptors. The frequency of 96% (for the aligned descriptor of *what teachers do*) was calculated based on the number of aligned activities divided by the total number of sample learning activities.

5.4.3 **Intention 2: Emphasis on reasoning**

As described in Chapter 3, the second intention, emphasis on reasoning, concerns students being able to explain a concept or solve a complex problem. Sample learning activities were analysed using the 3+1 framework as presented in Chapter 3 and reproduced in Table 5.22 to present the frequency analysis identifying strategies that encouraged deep reasoning by students.
As indicated in Table 5.22, just over half (51%) of the activities featured these thought-provoking activities for students to do in which they explored and constructed their own knowledge. An example of an activity which prompted reasoning is reproduced in Figure 5.6.

![Sample activity on reasoning.](image)

Some provision was seen for students to demonstrate reasoning when they explained why $\frac{17}{3}$ equals $5 \frac{2}{3}$. In this case, students were given freedom to draw any picture they felt appropriate to answer the question but with justification. Hence, in this activity students could be described as using the indicator explain their thinking and mathematical activity flexibly using written, spoken language, symbolic, iconic and enactive representations, thus aligning with the curriculum intention of allowing students to engage in multiple ways of expression to demonstrate their mathematics understanding. In the process, students may have opportunities to express their mathematical understanding through different modes of representation (enactive,
iconic and symbolic), particularly in iconic form (diagrams) showing why \( \frac{17}{3} \) equal \( 5 \frac{2}{3} \). Around 47% of such activities were designed and provided to students by teachers. However, there were more sample learning activities where students were encouraged to apply ready-made rules and formulae without understanding their origin (71%).

5.4.4 **Intention 3: Use of context**

Intention 3, use of context, is about incorporating meaningful contexts into a learning activity, which should be based upon students’ daily life activities or situations that they could readily imagine. Analysis of the 72 sample learning activities were analysed using the 3+1 rubric, which is presented in Table 5.23.

Table 5.23

*Frequency analysis on Intention Three (Use of Context)*

<table>
<thead>
<tr>
<th>Indicators</th>
<th>What the Teacher Does</th>
<th>%</th>
<th>What Students Do</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contextualise learning activities based upon students’ reality (daily-life activities or situations that can be readily imagined).</td>
<td>72</td>
<td>Actively engaged in solving context-based problems.</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>Counter indicators</td>
<td>Do not base learning activities on students’ reality, and instead rely upon use of de-contextualised mathematical learning activities.</td>
<td>28</td>
<td>Do not use their mathematical knowledge to solve context-based problems.</td>
<td>15</td>
</tr>
</tbody>
</table>

The findings indicate that most activities were designed around contexts related to students’ daily life activities, but were intended to be performed by the teacher (72%) rather than students (49%). For instance, one of the sample activities having these characteristics is re-produced in Figure 5.7.
Context-related objects were to be shown to students to demonstrate the concept of fractions from a whole object. The content of the sample was based on students’ daily life activities by using an apple and introducing the concept part of a whole. However, there were few activities students were meant to perform and use to construct new knowledge on their own. Instead, the activity clearly showed the intention of the teacher to demonstrate the concept of half by cutting an apple into two equal parts, followed by the teacher’s explanation. In this activity a teacher could be described as using a context indicator contextualise learning activities based upon students’ reality (daily-life activities or situations that can be readily imagined). The prevalence of such examples tends to indicate that most of the respondents are aware of the importance of designing meaningful, context-based learning. However, the majority of respondents seemingly lacked an awareness of engaging students in their own construction of knowledge and thus students could be described as using the counter-indicator do not use their mathematical knowledge to solve context-based problems.

In contrast, there were a few cases where students were intended to be involved in the activity. An example is as shown in Figure 5.8.
Students are to be provided with a context related task they can relate to their own experience of sharing among friends and siblings. At the same time, students are challenged with sharing two oranges equally among five friends, where they are made to think of the best method for doing that practically. In the process, students are encouraged to communicate their thinking and expression in terms of various modes (i.e., verbal, in words and in written). In this activity, students could be described as using the context indicator actively engaged in solving context-based problems.

5.4.5 Summary

Designing the sample learning activity was intended to provide respondents with an opportunity to share their intended classroom practice, upon which analysis could be conducted to determine the extent of alignment of the respondents’ planned practice with the intentions of the new curriculum. It was expected that the analysis of the intended learning activities would provide insights into the respondents’ beliefs regarding mathematics and mathematics education. The analysis has contributed to a comprehensive understanding of the beliefs held by those teachers who responded to survey questionnaires and who represented the overall population of Bhutanese mathematics teachers (refer to Table 5.5 and 5.6).

The analysis also suggests a discrepancy, or gap, between their expressed beliefs and those evident in their planned activities. Further, findings indicated some difference between the beliefs and practices between PTC and B.Ed in terms of alignment with the curriculum intentions. Compared to B.Ed graduates, PTC’s practices seem to align more closely with curriculum intentions.

Despite respondents’ general indication of a strong reform-oriented viewpoint in their expressed beliefs, their planning provided evidence of classroom pedagogy that seemed more aligned to Platonist beliefs. This belief is associated with more
explanation from teacher and less involvement of the students performing a task. This is in conflict with the intentions of the new curriculum, which focuses on engagement of students in learning tasks.

This claim is supported by the analysis of the sample learning activities with regards to the fourth intention of the curriculum, which promotes process standards. The analysis showed that the majority of respondents seemed to lack ideas for designing an activity in which students are encouraged to think, share and justify ideas based on their previous knowledge and experiences. Rather, most of the tasks tended to be procedural, involving a single solution approach and representation, with barely any discussion and communication of mathematical ideas. The activities appeared to lack provision for students to think and make sense of mathematics. Instead, learners appeared to be recipients of knowledge.

Further, analysis of the sample learning activities using the analytical framework indicated that the majority of the respondents were limited in their ability to design a learning activity for students which would promote relational understanding and mathematical reasoning, that is, the first two intentions of the curriculum. Compared to these first two intentions, most survey respondents were aware of the importance of using models, diagrams and concrete objects in designing a sample learning activity, as shown in Figure 5.6.

As illustrated in the Figure 5.6, the teacher intended to use an apple to demonstrate the concept of halves. However, such aids were again used by teachers in their explanations rather than allowing students to use these learning materials to help them understand the concept more deeply and be able to construct their own mathematical knowledge. Thus, analysis of the learning activities revealed that, despite their expressed beliefs that seemed aligned to the curriculum intentions, most teachers tended towards expository teacher-centred pedagogy rather than providing opportunities for students to use the process standards and which might inspire students to construct deep, relational understanding. The overall results seemingly indicated the existence of a gap between the intention and the implementation of the ideas in the new curriculum.
In summary, two findings that resulted from the analysis of the sample learning activities are as follows:

- The majority of sample learning activities provided by respondents lacked an emphasis on engaging students actively in doing mathematical activities; rather they were passive observers;
- The majority of sample learning activities emphasised procedural knowledge and so instrumental understanding (i.e., the ‘what’ of mathematics), rather than reasoning and the associated relational understanding (i.e., the ‘why’ of mathematics).

Based upon the sample learning activity evidence, it would seem that in general the respondents, demonstrated instrumental or Platonist beliefs regarding the teaching and learning of mathematics.

5.5 CHAPTER SUMMARY AND IMPLICATIONS

The findings from the this macro-level phase of the study were presented in four distinct sections: (a) demographic information of 80 respondents; (b) respondents’ beliefs about mathematics and mathematics education; (c) respondents’ beliefs and acceptance of the new curriculum, and (d) respondents’ sample design of learning activities on the given mathematical topic.

In Section 5.1, the respondents’ demographic information was presented and analysed. When compared to the latest Annual Statistics (Ministry of Education, 2013b), these data reflected the realities in Bhutan, in terms of the primary teacher population, gender and qualifications. This revealed that the sample of survey respondents was representative of the teaching population in Bhutan. Approximately 59% of the respondents had teaching experience ranging from one to seven years. These data confirm that more than half of the respondents had some exposure to the learning theories related to mathematics education and some orientation towards the standards of the new curriculum during their pre-service training. The teaching environment data suggested that the majority of respondents were from rural schools holding the position of classroom teacher, teaching mostly Grade 5 with class sizes of 31 – 40 students. The teachers teaching the largest classes size of more than 41 students were found to be in urban schools. The current trend of migration of the population from rural to urban is reflected in these findings.
In Section 5.2, the findings indicated that respondents showed strong support for experimentalist beliefs about mathematics and mathematics education, thus aligning with the new curriculum intentions. In section 5.3, respondents’ belief about the overall structure of the new curriculum were presented. It was found that the majority of respondents had a positive response towards the new curriculum, and thus were aware of the intentions of the new mathematics curriculum. This included those respondents with PTC qualifications who were not exposed to such ideas during their training period. It is noted that there was no significant difference found between PTC and Bachelor of Education qualifications in terms of their acceptance of the new mathematics curriculum. Hence, findings from this section tend to reveal that the majority of respondents expressed beliefs which were correspondingly aligned with the new curriculum.

Finally, Section 5.4 analysed the sample of learning activities designed by the survey respondents. The analysis was based upon the four intentions of the curriculum. First, the analysis focussed upon the fourth intention that relates to process standards. This analysis revealed that the majority of survey respondents did not comprehensively incorporate process standards into their sample learning activity designs. Of all the process standards (and, as presented in the analysis, the finer-grained sub-standards), incorporation of vertical connections was the most common among the respondents. Second, the sample learning activities were analysed with regards to the first three intentions of the curriculum using the 3 + 1 framework. The analysis showed that although the respondents seemed to be aware of the curriculum intentions, the majority of the learning activities designed were oriented towards teacher-centric explanation or demonstration rather than engaging students in the exploration and construction of their own knowledge.

In summary, the analysis of the survey responses during this macro-level phase of the study has begun to answer the first three research questions. These initial answers are presented as follows.

Question 1: What are the beliefs of Bhutanese primary teachers about mathematics teaching?

The findings from the analysis of teachers’ beliefs survey items suggest that the respondents generally expressed experimentalist beliefs. This is in contrast to the findings of the analysis of the sample learning activities, which suggest that the
respondents instead manifested Platonist beliefs. That is, what teachers claim to believe appears to differ from what they evidence in their planned learning activities, as indicated in Figure 5.10.

Question 2: What are Bhutanese primary teachers’ planning and classroom practices in teaching mathematics?

Most of the sample learning activities illustrated instructional approaches based on teachers’ explanation rather than engaging students’ in exploring and constructing their own knowledge. For instance, most of the learning activities were like a lesson plans rather than tasks for students to explore on their own. In addition to this, some of the samples were too brief to be termed learning activities, and without any clear instructions for students to carry out the task independently. Hence, the majority of respondents appeared to have adopted a traditional approach to planning and conducting mathematics lessons, as presented Figure 5.10.

Question 3: To what extent are mathematics teaching practices aligned with curriculum intentions?

In terms of planned practice, it was found that there were very few thought-provoking tasks evidencing all four intentions. The majority of the tasks were routine calculations requiring little thinking and few opportunities for students to apply process standards and thereby develop problem solving skills. That is, many of the sample learning activities suggested that teachers’ intended practices are not well
aligned with the intentions of the curriculum. Figure 5.11., illustrate the conflicting findings from the Section 5.2, of the survey, and the planned learning activities.

![Survey](image1)

**Figure 5.11.** Findings related to Research Question 3.

The overall results generated from the macro-level phase study suggest a gap between expressed and manifested beliefs, thus showing mis-alignment between intention and implementation of the new curriculum, as presented in Table 5.24

### Table 5.24
**Summary of Quantitative Findings**

<table>
<thead>
<tr>
<th>Source of the data</th>
<th>Findings</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Respondents’ beliefs about mathematics and mathematics education</td>
<td>Majority of respondents support constructivist view of teaching and learning of mathematics, thus aligning with intentions of the new mathematics curriculum.</td>
<td>There is a gap shown between the first two and the last set of data in terms of the relationship between expressed and manifested beliefs, thus creating mis-alignment between intention and implementation of the new curriculum. The majority of the respondents seemed to be aware of and strongly supported the belief statements associated with ideas intended in the new curriculum, but their learning activities suggested otherwise.</td>
</tr>
<tr>
<td>Respondents’ beliefs about the new curriculum</td>
<td>Majority of the respondents have a positive response to the ideas intended in the new mathematics curriculum.</td>
<td></td>
</tr>
<tr>
<td>Respondents’ design of sample learning activities</td>
<td>The majority of the respondents were unable to frame a learning activity aligning with the curriculum intentions.</td>
<td></td>
</tr>
</tbody>
</table>

There is limited evidence that the ideas and standards intended in the new curriculum guide teachers’ actual practices with students. The survey responses analysed at the macro level indicate that the majority of respondents expressed in favour of a child-centred approach. However, analysis of the sample learning activities provided by the respondents indicates that the actual practices of the teachers manifested a teacher-centred approach, suggesting that in reality teachers hold more transmission-oriented beliefs. That is, the actual practices and beliefs of Bhutanese primary school mathematics teachers are mis-aligned to the intentions of the new curriculum. The existence of a gap between the intentions and the implementation of the new
curriculum is clear, as presented in Table 5.25, and these contradictions required further in-depth qualitative investigation.

Table 5.25

**Summary of Alignment between Expressed and Manifested Beliefs**

<table>
<thead>
<tr>
<th>Curriculum Intentions</th>
<th>Expressed Beliefs (Survey)</th>
<th>Manifested Beliefs (Sample Learning Activities)</th>
<th>Alignment with Intentions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emphasis on relational understanding</td>
<td>Indicated</td>
<td>Some indication</td>
<td>Aligned</td>
</tr>
<tr>
<td>Emphasis on reasoning</td>
<td>Indicated</td>
<td>Not indicated</td>
<td>Not aligned</td>
</tr>
<tr>
<td>Use of context</td>
<td>Indicated</td>
<td>Some indication</td>
<td>Aligned mainly in the form of examples</td>
</tr>
<tr>
<td>Use of process standards</td>
<td>Indicated</td>
<td>Some indication</td>
<td>Aligned mainly with vertical connections</td>
</tr>
</tbody>
</table>

The need for further in-depth study was mainly due to the nature of data collected and analysed in the macro phase of the study. For instance, data collected from survey questionnaires may not be authentic in terms of the actual practice in the classroom. Pursuing this idea further, although there are some indications of practices being reflected in sample learning activities, the validity of findings could be strengthened further when seen in action. This idea tends to support and apply Bruner’s (1966) three forms of representation to identify a deep understanding of a mathematical concept. Moreover, of the three forms, the enactive form of representation is considered the most authentic. Hence, in this study, investigation of actual practice in the classroom is necessary to explore curriculum implementation. Results of this phase are presented and discussed in Chapters 6 and 7.
Chapter 6: Phase 2 Micro-level Context

As discussed in earlier chapters, this study was conducted in two phases. The first phase examined the beliefs and planning of teachers across Bhutan. This first phase, described as the macro-level study, was reported in Chapter 5. It was found that the majority of survey respondents expressed beliefs which supported the philosophical intentions of new mathematics curriculum. However, there was an apparent misalignment between respondents’ expressed beliefs revealed through the survey and their intended plans manifested in the design of sample learning activities. In contrast to the experimental beliefs evident in the survey responses, their planning reflected Platonist beliefs about mathematics teaching and learning. Therefore, the micro-level phase was undertaken to explore in greater depth this apparent discrepancy. This phase was designed to provide rich information that addresses the four Research Questions:

Question 1: What are the beliefs of Bhutanese primary teachers about mathematics teaching?

Question 2: What are Bhutanese primary teachers’ planning and classroom practices in teaching mathematics?

Question 3: To what extent are mathematics teaching practices aligned with the curriculum intentions?

Question 4: What influences the implementation of the curriculum intentions?

The micro-phase level of the study involved observing five teachers implementing a unit on fractions with Class 5 students. Data included video-recorded observations, the teachers’ associated lesson plans, teachers’ written reflections on their lessons, and transcripts of informal post-lesson interviews. Two teachers from Takin and three from Dragon participated.

The purpose of this chapter is to provide a profile of the context and participants. Results of the analysis of classroom observations and associated artefacts are presented in Chapter 7. This chapter begins by introducing each of the
schools and the teachers involved, in Sections 6.1. Section 6.2 provides an overview of the Unit (fractions) that was taught in all classes, and finally a chapter summary is presented in Section 6.3.

6.1 PARTICIPATING SCHOOLS AND TEACHERS

Takin and Dragon schools were chosen from the 40 randomly selected schools that participated in Phase 1 of the study. Convenience sampling was used based on the researcher’s proximity to the schools. Takin was chosen to represent an urban school and Dragon to represent a rural school. Both schools were located a relatively short distance from the researcher’s work place so regular site visits were possible. Brief details of each school are presented below, along with an introduction to the participating teachers from each school.

6.1.1 Takin

Takin primary school is situated on the outskirts of one of the larger towns in Bhutan and so is classified as semi-urban. The school was attached to a middle-secondary school, is co-educational, and had approximately 1000 students and 50 teaching staff. By Bhutanese standards, the classroom facilities were quite modern. At the time of the study the physical layout of classrooms featured benches and desks that were fixed in rows with a blackboard at the front facing the students. There was no carpet and hence the ambient noise level was high. There was some provision of extra equipment such as a white-board to help the teacher explain or demonstrate concepts during teaching. The walls were decorated with a few posters displaying some common mathematics rules. As is customary in Bhutanese classrooms, a small space in the corner of the classroom was reserved for a worship place to remind students to behave well both physically and mentally. A photograph illustrating this classroom is presented in Figure 6.1.

![Figure 6.1. Seating arrangement in Takin.](image)
As illustrated in Figure 6.1, the classroom was large enough to fit the 40 students but was not easily configured for group activities. At Takin, all lessons were of 50 minutes duration. There were two sections (streams) of Class 5 students and each had approximately 40 students, with a roughly equal number of male and female students. Each section was taught by a different teacher. The profiles of the two teachers, Phurba and Norbu, are described in the following sub-sections. These profiles are based upon the interviews, discussions and written reflections gathered before, during and after the observation period.

**Phurba**

Phurba was a male teacher with six years’ teaching experience across Class 4 - 10. At the time of data collection (2013), Phurba was assigned to teach mathematics to Class 5 students for the duration of the classroom-observation period. He had graduated with a Bachelor of Education in Primary Mathematics. As discussed in section 2.4.4, besides teaching other general subjects in primary level, a teacher with this qualification is specialised in teaching mathematics from Classes Pre-Primary to 10. Phurba was the only participant in the micro-level study to hold this qualification and was presumably exposed to constructivist-inspired principles in his pre-service course. Despite his university training and several years’ experience, Phurba revealed during one of the post-lesson conferences that, prior to participating in this study, he had never received any orientation about the intentions of the new curriculum and thus implemented the curriculum according to his own ideas:

> Although I have been teaching mathematics for good number of years, yet I didn’t have a clear understanding on how the new curriculum should be taught. Therefore, I taught mathematics without following the set principles and process standards intended in the new curriculum.

Having stated this, Phurba seemed to have found the new curriculum applicable to day-to-day life activities and not formula-oriented, suggesting he held experimentalist beliefs about mathematics and mathematics education. For instance, in one of the lesson reflections, Phurba confidently announced that he could assess students through some hands-on activities and observe students’ understanding being achieved. However, he noted that some students found the introductory problem-solving activities difficult, particularly reading and understanding the activity in English.
Norbu

Norbu was a female teacher who had been teaching mathematics in schools for the past six years. In this study, Norbu taught the second section of Class 5 students of Takin school for the duration of the study. She graduated with a Bachelor of Education (Secondary) and specialised in teaching mathematics. Although qualified to teach secondary students, she had also taught primary school mathematics for the past six years, including teaching mathematics to Class 5 students using the new curriculum. This experience motivated her to be one of the volunteers to take part in this study. Norbu believed that by participating in this study, she would benefit in terms of practice in teaching mathematics using the new curriculum. This motivation was reflected in a comment she made in her overall lesson reflection (see Appendix G):

..Although I have been teaching this new curriculum, I did not get a proper orientation to implement this new text to learners. As far as possible, I tried to seek help from the people who were oriented but that was not enough for me to apply practically to students. I didn’t know the actual motive and the objective behind this new curriculum..

According to Norbu, although she found the language part of the new curriculum the most challenging, particularly understanding the problems suggested in the textbook, she seemed to support the content of word problems as being connected to students’ daily life activities. However, in practice, as stated in one of her written lesson reflections, Norbu seem to be more focused on covering the syllabus in the allocated time, rather than on students’ quality of learning. She said, “Today I was able to cover the whole content with what I have planned” (Norbu’s reflection on Lesson One). The same point was repeated several times in her reflections during the study. She rarely commented on the level of students’ understanding of the topic. However, as she stated during one of the post-lesson interviews, she seemed to have realised that students learn better when they are engaged in doing activities: “when students were made to solve the problem on their own, they could do it themselves, so I feel that to make them understand more, we have to make them do the activities. Students learn by doing” (Lesson reflection 8).
6.1.2 Dragon

Dragon School is situated in a rural setting and was one of the oldest public schools in Bhutan, having been established in the 1960s. Compared to Takin, Dragon was relatively poor in terms of infrastructure and facilities. At the time of data collection, there were approximately 600 students at the school with 26 full-time teachers. Like Takin, the school was co-educational with almost equal gender representation and there were two sections of students in Class 5.

Despite the old buildings, the classrooms were quite spacious and the conditions were conducive to conducting group activities. Unlike Takin, the desks and benches were arranged in groups of six students rather than rows, as illustrated in Figure 6.2.

![Figure 6.2. Seating arrangement in the classroom (Dragon).](image)

The spaces between the tables allowed enough room for a teacher to walk around and interact with students. A few posters were found on the walls, each serving an instructional rather than aesthetic purpose. Three teachers were involved in the study, Sangay, Gawa and Tempa.

Like Norbu in Takin school, Gawa and Tempa volunteered to participate in this study, in addition to their regular teaching load. They taught fractions together to one section of students in Class 5, with the other section being taught by Sangay alone. Gawa and Tempa shared the six weeks of lessons in the ratio of 2:1 (Gawa: Tempa), because of their timetable. The scheduling of times they could teach was unpredictable over the weeks of the observations. Hence, Gawa taught in weeks 1, 2, 5 and 6 and Tempa weeks 3 and 4. Further, unlike in Takin, the researcher was challenged to find time to bring these three participating teachers together for the post-lesson interview. Many times, it was only the two teachers who taught that week...
who could attend. Similar to Takin, at Dragon, each section of Class 5 students took six 50-minute lessons to complete the unit on fractions. In the following subsections, each of the participating Dragon teachers is briefly profiled, including gender, qualification and teaching experience.

**Sangay**

Sangay was female, and like Norbu from Takin, held a Bachelor of Education (Secondary) qualification to teach mathematics. She had taught mathematics for seven years in various schools, including primary schools. At the time of data collection, Sangay was assigned to teach mathematics to one section of Class 5 students. Although she had previously taught Class 5 mathematics at other schools, it was the first time Sangay had taught Class 5 mathematics at Dragon. Sangay found teaching mathematics in Class 5 comfortable, as the content in the new syllabus was not as extensive as the previous curriculum. Moreover, unlike the previous curriculum, Sangay indicated that she found the new curriculum more interesting as it included a lot of activities to help arouse student interest (Lesson reflection 3).

According to Sangay, teaching mathematics was interesting and she tended to support the philosophical intentions of the new curriculum. Sangay appeared to be fully aware of the intentions of the new curriculum as quoted from the third collaborative post-lesson interview (3rd session) “compared to the old, the new curriculum is more interesting... it includes activities which arouse motivation in children to learn… children learn more by doing themselves”. Sangay expressed her enjoyment of teaching fractions in Class 5 using the different strategies suggested in the new textbook and guidebook. However, she appeared to show some concern regarding the problems being in English, which she found to be quite challenging for students. Hence, Sangay supported the ideas integrated in the new curriculum of *learning by doing*, provided students are equipped with the right materials.

**Gawa**

Gawa was a male and held a Primary Teacher Certificate (PTC) qualification received more than a decade ago. He had experience of teaching a variety of primary school subjects including mathematics, and both the previous and the new curriculum. With his experience of teaching both curricula, Gawa was an essential participant in this study. Among the five participating teachers, he was the only one who attempted to use an overhead projector in addition to using the blackboard. This
indirectly depicted his awareness of the NCTM principle of using technology to help students understand mathematical concepts. As one of the senior teachers at Dragon, Gawa was very busy with school co-curricular activities, in addition to his teaching hours. As a result, it was difficult to find sufficient time for Gawa to participate in the post-lesson interview.

**Tempa**

Tempa was the other male teacher with a PTC qualification. He had been teaching primary school since his graduation, which was in the same year and from the same Teacher Training College as Gawa. Further, like Gawa, Tempa had been teaching in Dragon school for approximately five years and he had previous experience of teaching Class 5 mathematics. Tempa appeared to be very willing to accept and adopt new ideas. For instance, when the researcher shared the concept of integrating exploration in the beginning of the lesson, he had his self-designed activity implemented in his following lessons. Moreover, Tempa appeared to be very sociable and passionate about teaching. His approach kept his students alert and active throughout the lessons, unlike the other participants.

### 6.2 UNIT LEVEL PLANNING

This section summarises the content (referred to as topics) of the Class 5 unit on fractions which was the subject matter taught during the micro-level phase. Before the content is discussed, the students’ anticipated prior knowledge is discussed in terms of what they would have been taught in previous years. Then, the set of topics covered in Class 5 Fractions Unit is summarised. Following that, the sequencing and timing of these topics as they were presented in the classes at each school are summarised.

As intended in the new curriculum framework, the basic concept of fractions is introduced from Class PP (Pre-Primary). For instance, in Class PP, the students are expected to understand simple fractions such as ‘halves and its meaning in context’. Here, students are expected to understand that parts of a fraction must be equal in size. Similarly, in Class 4, a concept of renaming fractions is introduced, along with the comparing and ordering of fractions.

In Class 5, students are expected to understand the meaning of fractions in broader forms such as renaming *fractions as divisions* by linking concrete to
symbolic with appropriate models and reasons. As a result, students are expected to represent fractions confidently in different modes such as enactive, iconic and symbolic (Bruner, 1966). Further, students are expected to have no difficulty in finding fractions of the same value in different forms, such as equivalent fractions. Having understood the meaning of equivalent fractions, students are expected to compare and order fractions more confidently and appropriately using fractions models and paper strips. Hence, at the end of the unit, students are expected to have mastered the meaning of fractions, fractions as division, represent equivalent fractions in different modes, and be able to order fractions.

In Class 5, the fraction unit is presented as a set of four topics in the students’ textbook and teachers are provided with the Teacher’s Guidebook to help teach those topics (details in Chapter 2). The topics are briefly summarised in the next Section 6.2.1. Following that, the sequencing of these topics in each class over the observation period is summarised.

6.2.1 Topics

In this section each of the topics in the Class 5 fractions unit is presented, in the order that the curriculum suggests.

**Topic 1: Meaning of Fractions**

In this topic, teachers are expected to explore students’ basic knowledge about fractions. For instance, students are exposed to different approaches for deriving a meaning of fractions (e.g., \(\frac{3}{4}\)) such as: \(\frac{2}{4}\) can mean:

- \(\frac{3}{4}\) of shape
- \(\frac{3}{4}\) of a group
- \(\frac{3}{4}\) of a length

**Topic 2: Fractions as Division**

In this topic, teachers are expected to help students explore the concepts of fractions in different forms such as meaning of fractions as division (e.g., \(\frac{3}{4} = 3 \div 4\)) connecting fractions with previous knowledge on the concept of division. In the process, students are exposed to different types of fractions such as improper fractions and mixed numbers.
**Topic 3: Equivalent Fractions**

In this topic, students are expected to explore the meaning of equivalent fractions using different methods.

**Topic 4: Comparing and Ordering of Fractions**

Under this topic, students are expected to have achieved a deep understanding of the concept of fractions and be confidently able to identify the value of any type of fraction. For instance, given any pair or set of fractions, students are expected to be able to compare and order them using multiple methods.

### 6.2.2 Sequencing

The sequencing and number of lessons taken to complete each topic differed across the two schools and four classes. The number of lessons spent on each is summarised in Table 6.1.

**Table 6.1**

*Topic Sequencing and Number of Lessons Taught*

<table>
<thead>
<tr>
<th>Topic</th>
<th>Takin</th>
<th>Dragon</th>
<th>Phurba</th>
<th>Norbu</th>
<th>Sangay</th>
<th>Gawa</th>
<th>Tempa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meaning of fractions</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Fractions as division</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equivalent fractions</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td></td>
<td>2</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Comparing of fractions</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ordering of fractions</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total lessons</strong></td>
<td><strong>8</strong></td>
<td><strong>8</strong></td>
<td><strong>6</strong></td>
<td><strong>4</strong></td>
<td><strong>2</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Although the five teachers taught the same number of topics, the number of lessons taken for each topic differed. For instance, participating teachers in Takin took four more lessons in total compared to Dragon. Moreover, all the teachers appeared to have taught following the same sequence of topics except for Phurba, who repeated the topic *fractions as division* in his fifth lesson. As shared during the fourth post-lesson interview, the reason for repeating the topic was due to Phurba’s realisation of mistakes in his previous lesson.
6.3 CHAPTER SUMMARY

In this chapter, the context of the micro-phase level study was presented with a brief description of each of the following:

- Two chosen schools (Takin and Dragon)
- Participating teachers (Phurba, Norbu, Sangay, Gawa and Tempa)
- Unit on fractions
- Topic sequencing and number of lessons.

This information is provided to ensure understanding the participants and then contexts in Phase 2. In keeping with case-study design, multiple sources of evidence have contributed to depicting the context of this study and the tasks that the participants were expected to implement as part of their teaching. Having presented the context of the micro-level study in this chapter, the results of the data analysis and subsequent findings are presented in the following chapter.
Chapter 7: Phase 2 Micro-Level Results

The context of the micro-level phase was introduced in Chapter 6, and this chapter presents the results of the analysis of the data gathered during this phase. The chapter is presented in two main sections, each divided into further sub-sections. Section 7.1 addresses the analysis of the lesson plans submitted by the participating teachers for the lessons they taught. In this Section, the lesson plan analysis is presented based on four lesson components as the curriculum recommended instructional design.

Section 7.2 presents the analysis of the observed lessons using the 3+1 framework developed in Chapter 3, in order to explore the alignment of the practices with the four curriculum intentions. In Sections 7.2.1, the analysis reports the teaching practices observed in the first topic taught and the alignment of the teachers’ lesson plans with the actual teaching of the lessons. The purpose for choosing the first topic (on the meaning of fractions) was to understand the existing practices of participating teachers without any intervention from the researcher. Based on the claims presented in Section 7.2.1, Section 7.2.2 presents an analysis of the remaining 24 lessons taught by the five participating teachers. The results of the lesson observations were supplemented by analysis of the post-lesson interviews. The chapter concludes with a short summary in Section 7.3.

7.1 ANALYSIS OF SAMPLE LESSON PLANS

In this section, the results of the analysis of one of the 28 lesson plans submitted by the participating teachers are presented. The sample lesson plan presented is Phurba’s first lesson on the meaning of fractions. It was chosen for two reasons: (a) this lesson plan was the first lesson observed by the researcher and video-recorded and hence reflected ideas less influenced by the researcher, and (b) Phurba was the most qualified of all participating teachers for teaching mathematics at primary school levels, as discussed in Chapters 2 and 6, and hence might have been expected to present an optimal plan. Phurba’s lesson plan is presented in Figure 7.1 and can be seen to comprise four components.

- Lesson objectives → Component A
- Information input → Component B
- Activity ➔ Component C
- Closure ➔ Component D

Figure 7.1. Sample lesson plan.
The sample lesson plan defines two lesson objectives (Component A) regarding what a child should be able to do by the end of the lesson: (a) represent fractions using shape, length and group correctly and (b) to write a fraction for a given picture. Subsequently, the information input presented in the sample lesson plan is aimed at achieving those concerned objectives. All the intended questions are closed questions with no indication that any discussion or elaboration on the answers will be encouraged. This information input section appears to indicate the role of the teacher is to transmit knowledge with little discussion.

The information input (Component B) begins with some revision with closed questions to establish the students’ prior knowledge. Although not indicated clearly, to achieve the first lesson objective, it is assumed that a teacher has planned to present an example on each of the following concepts for the students during the information input:

- **Representation of fractions using group** is indicated with a group of seven students, of whom three are boys (3 out of 7 as \(\frac{3}{7}\)) and four girls (4 out of 7 as \(\frac{4}{7}\));
- **Representation of fractions using shape** is illustrated with a circle divided into eight equal parts (not exactly equal in parts) and three parts shaded to indicate 3 out of 8 as \(\frac{3}{8}\); the same idea was attempted to be imparted using a realistic but imaginary example of an apple cut into four equal parts and one part taken away to show 1 out of 4 as \(\frac{1}{4}\) parts (context);
- **Representation of fractions using length** is shown by drawing a line divided into eight equal parts to illustrate a distance in kilometres covered by different modes such as walking and by car. As marked, 5 kms are covered by walking (5/8) and the remaining 3 kms by a car (3/8).

This analysis of the plans provides a strong indication of awareness by the teacher regarding curriculum intentions, particularly the first (i.e., emphasis on relational understanding) and the third (i.e., use of context). For instance, the teacher has cited examples using various approaches to help explain the targeted concept. In the process, examples cited are mostly context-related, such as using students themselves (to explain a concept of fractions using groups), cutting up an apple (into four equal parts) and distance covered in kms (by walking and by car). From these
activities, the teacher tended to assume that students were provided with ample opportunity for achieving a deep understanding of the meaning of fractions.

Having intended to deliver this content through explanation, the plan suggests that teacher would then proceed with a series of activities (Component C) to evaluate understanding of the information input. The set of activities appear to be aligned with the information input, and were based on the lesson objectives. For instance, the first task was to draw a picture to represent the given fractions (i.e. 2/3, ¼, 2/5 and 1/3). In this task, there was no specific guidance given to students in regard to the representations of fractions. It appears that students were to be given freedom to draw using any approach to represent fractions. Students could represent fractions either in the form of a part of a whole unit (e.g., a part of a whole apple) or a part of whole set (e.g. a member of a group of children). This task aligns with curriculum intentions, particularly with regard to the first intention’s emphasis on understanding, where students are given the opportunity to identify and describe the development of horizontal connections (drawing pictures related to students’ experience) and vertical connections (symbolic to iconic form).

Similarly, the same curriculum intention is evident in the second task when students are asked to write a fraction for each given picture, which requires students to represent the fraction in symbolic form. While students were performing these activities, the teacher planned to move around to check whether they were able to achieve the outcomes as explained earlier during the exposition component of the lesson. There was no specific activity allotted to summarise the lesson; instead, the teacher has merged the students’ activity with the lesson closure (Component D).

In this sample lesson plan, Phurba has presented various strategies to introduce the concepts, and make connections both horizontally and vertically, so aligning with the curriculum intentions. However, there was a very limited provision of thought-provoking tasks to encourage students to explain their thinking and mathematical ideas. Further, the use of context was limited to citing context-related examples and not for students to solve a contextualised task, so developing their problem-solving skills, as encouraged by the fourth intention. Moreover, the examples in the lesson plan appeared to be relatively trite without complex depictions of situations that would support problem solving. That suggested that the teacher may have been aware but not considered it important. As presented in the preceding paragraphs, out
of the four curriculum intentions, the sample lesson plan indicated that the teacher seemed to have some awareness of all but the second intention (emphasis on reasoning), which was mostly absent. The remaining 27 lessons are summarised and presented in Appendices E1-E5.

Phurba’s first lesson plan was compared to the other 27 lesson plans and it was noted that all the remaining lessons appeared to have followed a similar structure composed of four main components: objectives, information input, activity and closure. The lesson plan format adopted by these five teachers still followed the age old template introduced to them when they were student teachers in teacher-training colleges. All the five participating teachers, at least with regard to their planning, appeared to hold behaviourist views of teaching and learning of mathematics reflecting Platonist beliefs. The teachers’ plans tended to exhibit authoritative discourses or styles of teaching on which it was assumed that knowledge is transferrable. Support for this interpretation of the plans draws on the following four observations.

First, the lesson formats stipulated lesson objectives to be achieved at the end of the lesson, which is to be measured by the lesson closure. Second, the flow of the lesson appeared to be direct and linear. For instance, information input appeared to depend on lesson objectives and the learning activity is provided to evaluate the information input delivered. Likewise, lesson closure was intended to summarise and evaluate whether students had achieved the set lesson objectives. Third, this approach to planning is in contrast to the lesson components suggested by the new curriculum, which support a constructivist view. According to the new curriculum, a lesson is intended to begin with what students already know on the targeted topic, rather than a top-down or transmission approach of must-know based on the framed lesson objectives. Fourth, although there is some indication of curriculum intentions in the sample lesson plans, particularly the first (emphasis on understanding) and third (use of context), implementation appears to be limited. Thus, despite awareness of the curriculum intentions, in terms of practice, participating teachers seem to believe in explaining the concepts and ideas more than engaging students in constructing their own knowledge through discussion and engaging activities.

A few important issues were identified from the learning activities in terms of beliefs about mathematics and mathematics education. As observed, most of the
learning activities were transmissive with little thinking required from students. Moreover, during the exposition phase of the lesson, there was evidence of the teacher adopting explanation strategies rather than letting students explore, discuss and construct their own knowledge through making connections between new concepts and already known concepts. Based on the intended practice indicated by the sample lesson plans, it appears that teachers were more inclined to support Platonist than experimentalist beliefs about mathematics and mathematics education. The content of the lesson plans were limited in terms of alignment with the intention of the new curriculum. To complement the analysis of the teachers’ lesson plans, an analysis of the actual conduct of the lessons is presented in the next section.

7.2 CLASSROOM PRACTICE

Although lesson plans give some insight into intended practices, actual implementation may not be fully captured by the plans. Thus, it was necessary to explore the actual conduct of the lesson to understand whether a lesson was implemented consistent with the lesson plan, and so whether the actual teaching and learning reflected Platonist beliefs. Section 7.2.1 analyses the observations of the teaching of the first topic in each of the four sections of Class 5 students. This analysis focusses primarily upon Phurba’s teaching, which is compared to the three other teachers, because he was the only teacher who had a qualification in teaching primary mathematics with some mathematics teaching learning theories background.

The results are presented in terms of the four lesson components suggested by the new curriculum. Each of the lesson components is further analysed with the help of the 3+1 analytical framework. The purpose of this analysis was to explore the alignment of the actual lesson delivery with those four components suggested and to confirm the analysis of the lesson plans, prior to any influence from the researcher. The analysis begins with a short description of how Phurba conducted his first lesson. Following that, a comparison with the other four participating teachers is provided. In Section 7.2.2, the remaining 24 observed lessons are analysed. Finally, in section 7.2.3, selected examinations papers are reviewed and analysed further to inform the findings from the lesson plans, observations, and post-lesson interviews.
7.2.1 **Lesson Analysis of Topic 1**

This section presents the analysis of the first lesson on the topic (meaning of fractions) taught by the four teachers in their respective classes. This analysis begins with a short description of how Phurba conducted his first lesson, which is then compared with the lessons of the other three teachers. Phurba began his first lesson with a short closed questioning session based on the concept of fractions [what is “the concept of fraction”? studied in the previous class levels (e.g., Pre-Primary - 4). Then, during the exposition phase, he invited a group of seven students including three girls to the front of the class to help him explain the concept of fractions in terms of girls in the group as 3/7 (three girls out of seven students) and a fraction of boys as 4/7 (four boys out of seven students). With this presentation, Phurba intended to explain the concept using fractions as a part of a whole set.

Besides demonstrating the idea of representing 3/7 (girls) and 4/7 (boys) enactively, he also tried to represent the idea through drawing diagrams to show the same information iconically. Moreover, keeping the same concept, he then presented an imaginary story about a father trying to share an apple equally among his four sons, as reproduced below:

> A father gave one apple to his four children to share equally among themselves. Karma, one of the sons has eaten his share of an apple. So, how many parts are left?

The intention of the teacher here was to introduce the concept of fractions using an approach to a part of a whole unit, as shown in Figure 7.2. This seemingly indicated an attempt to integrate another context-related example into his teaching. Phurba’s practices of repeating several examples for the same concept indicated that he was aware of the importance of using different examples but it is not clear what benefit it would bring, given that students did not have a chance to explore the activity on their own. Phurba provided limited opportunity for the students to explore and identify the development of vertical and horizontal connections. Phurba then proceeded by explaining some more information to students, as shown below:

- Can a fraction be a part of a shape? (explained with the help of diagram and its shading: 5 shaded out of eight equal parts is 5/8) as shown in Figure 7.2.
Can a fraction be a part of length? Explained with the help of drawing a line to represent a distance as shown in Figure 7.3; a student walked 2 kms out of 3 from home to school to represent the distance walked as the fraction 2/3.

Including the illustrations in Figure 7.2 and Figure 7.3, Phurba completed his introduction of fractions as a part of group, a shape and a length. In the process of explanation, Phurba attempted to relate examples connecting not only vertically within mathematical ideas but also horizontally to students' life activities. However, although Phurba did his best to explain the concept using various approaches, it was difficult to ascertain whether students really gained understanding of the meaning of fractions.

During what appeared to be the formative assessment phase of the lesson, Phurba invited a few students to represent $\frac{3}{7}$ in diagrams on the board, which was followed by a list of low-level questions for the whole class to solve individually. He ended the lesson without referring to any of the suggested questions from the Practice and Applying section of the textbook. Thus, in terms of alignment with the four lesson components suggested in the new curriculum, none were completely demonstrated in this lesson as intended in the curriculum.

However, although the lesson was presented using a teacher-centred approach, there was some indication that Phurba tried to use different techniques to explain the concept of fractions. Having done this, there is an indication of Phurba emphasising understanding of the concepts, although he explained everything, rather than
providing contextual tasks for students to construct meaning on their own. Such findings indicate that Phurba was limited in designing an appropriate task to engage students constructively in helping them to develop relational understanding. In this sense, perhaps Phurba did not perceive this as an effective way to teach mathematics.

Norbu’s practice was similar to Phurba’s except in the way the lesson topic was introduced. Unlike Phurba’s lesson, Norbu used three authentic objects (three identical but different coloured lunch boxes) to introduce the concept of fractions. For example, two out of three lunch boxes were red and so represented $\frac{2}{3}$, and one lunch box was blue and so represented $\frac{1}{3}$. This particular idea of using real objects as an example aligns with both NCTM’s process standards (e.g., connection) and RME principles (horizontal connection). Hence, in this activity, Norbu can be described as evidencing the context indicator, *contextualise learning activities based upon students’ reality (daily-life activities or situations that can be readily imagined)*.

In addition to this example, Norbu presented further approaches to introduce the concept of fractions, which are summarised as follows:

- Showed picture of fruits to introduce fourths
- Drew number line to explain the concept of fifths.

Norbu seemed to be aware of the importance of knowing different approaches to introduce the mathematical concept during the exposition phase. Having exposed students to these examples, Norbu’s teaching can be described using the understanding indicator connecting both *horizontally between students’ reality and vertically between mathematical ideas of varying levels of abstraction or sophistication*. However, all the activities were carried out to help her explain, not for students to use in constructing their own knowledge.

While using these approaches such as pictures of fruits, fraction models and lunch boxes, Norbu was actually aligning her practice with the curriculum intentions, particularly the first and the third. For instance, while using lunch boxes, a group of students, and pictures of fruits, her ideas was not only to connect with realities (use of context), but also to help students understand the concept, although through the teacher’s explanations. Like Phurba, all these techniques were used for the teacher to
help explain the concepts and not for the students to develop a relational understanding.

In this way, there were limited opportunities for students to work on their own, describing students’ understanding indicator as actively participate in assigned task, in which they identify and develop horizontal and vertical connections within their own conceptual schema. Instead, they remained passive recipients. This result suggests that Norbu was well equipped with knowledge of various approaches to introducing fractions; however, she seemed to be seriously limited in terms of engaging students in learning by doing, thereby limiting students in connecting their own understanding of knowledge both vertically and horizontally.

Like Phurba and Norbu, Sangay began her lesson with closed questioning to introduce the topic. However, in implementing the exposition component, Sangay’s approach was slightly different from that of the other teachers in terms of delivery, as she used paper folding, to explain the concept of fractions. This explanation session was followed by a group activity on folding paper to represent fractions. Finally, she invited a group representative to present the outcome to the whole class. Nevertheless, the nature of the activity was limited in encouraging students to engage in deep and productive conversations as intended in the curriculum. There was a lack of clear instructions on how to carry out the group activity using one piece of paper provided in groups. Hence, the activity enabled only one individual to perform it: the rest of the students remained idle spectators. This is another limitation in practice, in which the majority of students could be described as using the reasoning counter-indicators, do not flexibly use a range of language and symbolic/iconic/enactive representations. Although Sangay seemed to value group activities to promote thinking and sharing of ideas, as intended in the curriculum, she did not appear to have the skills or materials to orchestrate a collaborative activity.

Further, like Norbu, to implement formative assessment, Sangay ended her lesson by asking the class to define the meaning of fractions which she herself explained based on a few responses received from students. However, she was the first of the observed teachers to encourage students to practise and apply what they were learning. As intended in the new curriculum, the conduct of Practice and Applying is to help enhance students’ understanding of the concept discussed during the lesson. Nevertheless, her approach did not differ markedly from those of Phurba
and Norbu. It was limited in helping students gain a relational understanding, despite her inclusion of a group activity to promote communication.

In his first lesson, Gawa had an interesting way of introducing the lesson with a thought-provoking problem: “I have one A4 size paper and want to give it to four students. What should I do to make everyone happy?” He challenged students with questions such as “What should I do?”, “How shall I share?” and “Shall I make it half?” These types of questions were expected to prompt students’ thinking and reasoning, thus aligning with the second curriculum intention. This particular problem appeared to align with the suggested exploration in the beginning of the lesson, integrating with the four curriculum intentions.

For instance, the question “what should I do to make everyone happy?” aligns with the third intention of using context to help students to think and share their ideas, and at the same time, develop relational understanding. In this activity, Gawa could be described as using the reasoning indicator use thought-provoking learning activities which encourage students to explain their thinking and mathematical ideas. However, this problem was short lived, as students were not given adequate opportunity to think further: students did not get the chance to solve the problem and so students could be described using the reasoning counter indicators, do not flexibly use a range of language and symbolic/iconic/enactive representations. Gawa presented the activity without actually allowing students to solve it and then moved on to his next activity, which included the demonstration of folding paper, to help introduce the concept of fractions.

The overall impression from these four teachers in their first lesson on fractions suggests that they were quite aware of the curriculum intentions. This reflects the findings from the macro-level phase of the study. However, in practice, almost all of them seemed to have problems implementing these intentions. For instance, there were several context-related examples used to help the teacher explain the taught concept rather than providing a contextualised task for students to actually do and thereby construct their own knowledge. As discussed in Section 7.1, it appeared that the teaching approaches adopted were geared more towards teacher-centred transmission, rather than student-centred construction. In this way, the provision of context-based problems used consistently for students to explore and be actively involved was severely limited.
Moreover, in the class, teachers were found to spend most time explaining the concept and tended to focus on the practice of exposition as shown in Table 7.1. The information in the table is provided to clarify further the practice of teachers in terms of alignment with four lesson components. This frequency table presents an analysis on Topic One in four of the Class 5 students in two schools as a sample model for the rest of the lesson plans.

Table 7.1
*Activity Type Frequency Table of the First Lesson*

<table>
<thead>
<tr>
<th>Teacher/Lesson Activities</th>
<th>Exploration</th>
<th>Exposition</th>
<th>Formative Assessment</th>
<th>Practicing and Applying</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phurba</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Act 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Act 2</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Act 3</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Act 4</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Act 5</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>Norbu</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>Act 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Act 2</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Act 3</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Act 4</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Act 5</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>Sangay</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Act 1</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Act 2</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>Act 3</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
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<tr>
<td>Act 4</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>Act 5</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>Gawa</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Act 1</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
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<tr>
<td>Act 2</td>
<td></td>
<td></td>
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<td>√</td>
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<td>Act 3</td>
<td></td>
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<td>√</td>
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<tr>
<td>Act 4</td>
<td></td>
<td></td>
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<td>√</td>
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<tr>
<td>Act 5</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>Act 6</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
</tbody>
</table>

The ticks in the table represent instances of activities in each lesson that have been classified as one of the four lesson component types. Under each of the lessons taught by the respective teachers, there is a list of activities presented to indicate a number of different activities intended in a lesson. As indicated in Table 7.1, of the four, exposition dominates each of the four lessons. These results seem to contradict the intentions of the new curriculum, where exploration is expected to dominate practice in the mathematics classrooms. The analysis of the first lesson taught by the first four participating teachers tended to present the overall picture of general practices adopted by existing Bhutanese teachers. Consequently, these findings tend
to re-confirm the claims presented in Chapter 5 and also in section 7.1, that the teachers’ held Platonist beliefs about mathematics and mathematics education. The remaining 24 observed lessons were analysed in regard to the activity types, presented in the next section.

7.2.2 Activity-type level analysis through the four lesson components

This section presents the analysis of the 24 remaining observed lessons. As in section 7.2.1, the researcher analysed the teachers’ practices in terms of implementation of curriculum intentions using the 3+1 framework. The reason for this approach was to explore the extent to which their practice was based on their beliefs discussed in Chapter 5. To help analyse the teachers’ practice in the classroom, the four lesson components (exploration, exposition, formative assessment, and practice and applying), suggested by the new curriculum, were used.

Prior to the introduction of a new concept, teachers are expected by the new curriculum to explore students’ existing understandings through a context-based task (e.g., Try This activity A). In so doing, as argued by An et al. (2011), a strong sense of personal innovation journey is needed in order to assist students to explore and construct meanings on their own, and with other students. In this way confidence in learning mathematics is likely to be enhanced. The depth and quality of students’ performance in the exploration phase is expected to inform the teacher about the type and level of information (exposition) to be delivered. For instance, prior to the introduction of fractions as division, a teacher is expected to explore students’ levels of understanding on basic concepts of fractions. Once students are exposed to the new concept, their understanding level is expected to be assessed by conducting formative assessment. The intention of the Try This activity B as discussed in Chapter 2, Section 2.3.1, is suggested as a follow up of the previous Try This Activity (main). Students are expected to enhance their understanding level by being encouraged to solve some of the related exercises from the last component of the suggested lesson plan (practice and applying).

Further, a small activity in the Unit Revision section is suggested in the student textbook to assess students’ understanding of the relevant mathematical concept in summative form for the purpose of recording students’ performance. Hence, if all these suggested components are followed consistently, the curriculum is designed to
help students’ gain relational understanding and thus derive deep conceptual knowledge.

To provide in-depth information about the prevailing practice of the participating teachers, some of the key examples which have integrated in terms of both suggested lesson components and curriculum intentions are summarised in Table 7.2. Of these three examples were identified from Phurba’s lessons, one lesson exemplified the lesson component (exploration) and two exposition. In this case, none of Phurba’s lessons incorporated either of the other two components (Formative Assessment and Practice and Applying).

Table 7.2
Sample of Practices identified from the Participating Teachers

<table>
<thead>
<tr>
<th>Lesson components</th>
<th>Phurba</th>
<th>Norbu</th>
<th>Sangay</th>
<th>Gawa</th>
<th>Tempa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploration</td>
<td>Lesson 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exposition</td>
<td>Lesson 3 &amp; 7</td>
<td>Lesson 1</td>
<td>Lesson 2</td>
<td></td>
<td>Lesson 1</td>
</tr>
<tr>
<td>Formative assessment</td>
<td>Lesson 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Practicing and applying</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As indicated in Table 7.2, a total of nine lessons were identified to help explain the implication of the four lesson components and interaction suggested in the new curriculum. Of the nine, there were three that contained some of features of the component (exploration), whereas Formative Assessment was present in one and Practising & Applying not evident at all. This was another indication of the teachers’ avoidance of using some of the activities/tasks suggested in the textbook based on the intention of the new curriculum.

Moreover, examples in which both suggested lesson components as well as curriculum intentions were integrated were few. One example was Tempa’s design of learning activity in his second lesson to help students explore the concept of equivalent fractions. This activity allowed for exploration and also integrated almost all the curriculum intentions when analysed using the 3+1 framework. The details on each these examples are now presented within the four suggested lesson components.
Exploration

According to the new curriculum, the first component of a lesson was expected to be an activity for students to explore and construct new knowledge on their own, connecting both horizontally and vertically with previous and subsequent learning activities. The context-related learning task was suggested to take place prior to the introduction of the new topic. The main purpose of this task is to help students reveal their prior understanding of the topic so that the teacher can gain a better sense of how to conduct the next activity, thus connecting vertically.

A common practice observed was that the participating teachers began most of their lessons by revisiting a previous lesson through low-level questions. For instance, while conducting his first lesson on the second topic, Phurba began with a question “What is a fraction?” When this question generated no response from the class, Phurba continued his questioning, asking “alright, fraction is a number [pause]. Do you know how to write fractions? Anyone want to try?” Phurba continued the same type of practice until the fourth lesson, for example, as reproduced from Phurba’s Lesson 4:

Teacher: What did you learn in the last class?

Students: Equivalent fractions (in chorus)

Teacher: What is an equivalent fraction?

Student 1: Equivalent fraction is a same

Teacher: Do you agree with her? Yes or No? Yes, equivalent fractions is representing the same amount of shape.

In a sense this was a kind of exploration practice but relied on a range of simple revision questions directed at presumably randomly selected students. Following a low-level closed questioning session as presented above, students were not given opportunities to connect their mathematical knowledge horizontally and vertically. In this activity, students could be described as evidencing the understanding counter indicator, does not describe the vertical and horizontal connections they perceive between mathematical ideas. Moreover, as indicated in the last question, the teacher answered his own question rather than letting students answer, which could be described as using the understanding counter indicator use of learning activities which do not incorporate opportunities to explicitly explore the connections between
mathematical ideas. Thus the common practice of beginning lessons with questioning was poorly aligned with the intentions of the exploration lesson component and with all of the curriculum intentions described in Chapter 3.

The second common practice observed during the exploration phase of the lesson was the way the Try This activity was conducted. According to the new curriculum, the Try This activity was meant for students to explore their understanding of previously taught mathematical concepts as a foundation that will shape their understanding of the new topic. However, none of the participating teachers were observed to use the Try This activity in this way. Instead, on the few occasions that the Try This activity was observed, it was carried out through the teacher’s explanation or conversation rather than by letting students explore on their own. For example, Phurba’s second lesson was conducted as suggested in the textbook (“Bijoy cooked a pot of soup for $1 \frac{1}{4}$ hour. She stirred it every $\frac{1}{4}$ hour. How many times did she stir the soup?”). He also used another example: “Imagine you have two apples and asked ‘how many fourths are there in total?’” He used this question to help him explain the concept of fractions as division. He tried to explain this question with two circles drawn on the board to represent the two apples, as illustrated in Figure 7.4. Both the circles were divided into four parts, shaded one whole and one-fourth of the other circle to help students understand the concept of five fourths.

![Figure 7.4. Sample diagrams drawn by Phurba.](image)

The way Phurba tried to explain the meaning of ‘fractions as division’ using his own apples example indicated that he was aware of the importance of connecting horizontally between students’ reality and mathematical ideas. At the same time, using the process of cutting apples in fourths, Phurba connected vertically between
mathematical ideas of varying levels of abstraction or sophistication from \( \frac{1}{4} \) of an apple to \( \frac{1}{4} \) of an amount of time considering the concept of time in quarters is more sophisticated than the concept of an apple in quarters. In this activity, Phurba can be described as using the understanding indicator of scaffolding connections horizontally between students’ reality (i.e., daily-life activities or situations that can be readily imagined by the students) and mathematical activities, and vertically between mathematical ideas of varying levels of abstraction or sophistication.

However, in endeavouring to explain the whole activity and not letting students explore on their own, Phurba demonstrated his limitations in terms of understanding the actual purpose of the Try This activity as suggested by the new curriculum.

Initially, Phurba was not keen to conduct the Try This activities and developed the practice of leaving the task incomplete and moving on to the next activity. During the first post-lesson conference, the researcher inquired regarding the use of the Try This activity, Phurba responded (Appendix G):

Madam, is it necessary to discuss the answer for the Try This inside the class? Can’t we leave it half way through and move on to the next, I mean information input? I mean, just touch a bit and move on to the information put…madam, my problem here is about the time constraint in completing the activity in one period. I am sure, in order to complete solving it, it will take more than a period and we will not be able to cover the rest of the plan if we stick to Try This alone (1st post lesson interview).

Rather, as indicated, Phurba’s intention was too focussed on covering the syllabus in time rather than on doing all of the recommended activities. Similarly, Norbu commented that she seldom used the Try This activity as it “takes a lot of time”, explaining that the students find it difficult to understand the problem since it is written in English. In contrast to the philosophy of the new curriculum regarding the contextualisation of a mathematical task, mathematics in Bhutanese schools is taught in English, and therefore there is a limitation in having resources such as the story boards, situations or examples in Bhutanese. The majority of the participating teachers had not attended orientation program on how to implement the new curriculum. At best, they seem to have received a school-based workshop, in which some of the ideas of the curriculum were presented in diluted form. For example, Phurba, when asked about the limited use of the Try This activity, claimed that
during the school-based workshop he was instructed to use the Try This activity only if time permitted.

However, there were exceptions to the routine practices observed as exploration activities. The first example was the paper-folding activity observed in Gawa’s second lesson, previously discussed in Section 7.2.1. In this lesson, Gawa was observed posing thought-provoking questions. Unlike in the first lesson, Gawa extended the activity for the students to explore, rather than using his own explanation. For example, his initial question was, “I have three A4 sized papers and wanted to share them equally among four students”. He then challenged students with questions such as “What should I do? How shall I share? Shall I make it half?” These types of questions were expected to prompt students in thinking and Gawa can be described as using the reasoning indicator *use thought provoking learning activities which encourage students to explain their thinking and mathematical ideas.*

In the following, students were given the opportunity to work on their own, linking iconic to symbolic representations by drawing three identical shaped rectangles, each to be divided into four equal parts and then converting the diagrams into fractions in symbolic mode, as illustrated in Figure 7.5. In this way, students were encouraged to have interpreted it as sharing three rectangles among four students, with each receiving one quarter from every rectangle to get three quarters in total ($\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ quarters). In this activity students could be described as using the reasoning indicators *explain their thinking and mathematical activity through flexible use of symbolic, iconic and enactive representations.*

\[ \frac{3}{4} \text{ as } 3 \div 4 \]

*Figure 7.5. Gawa's sample learning activity from lesson 2.*
Further, students were encouraged to connect their mathematical ideas horizontally with daily-life activities, or a situation that could be readily imagined, and vertically with varying levels of abstraction or sophistication. In this activity students could be described as using the understanding indicator, *participate in activities in which they identify and describe the developing horizontal and vertical connections within their own conceptual schema*. However, instructing students to choose three rectangles rather than any three whole objects limited the freedom in children’s thinking.

The second example is cited from Tempa’s lesson. This activity was a self-designed task presented for students to solve, which is reproduced as follows:

There were two boys, Pema and Tshering. Their mother gave them two packets of biscuits each containing eight pieces. Pema ate $\frac{1}{4}$ of the packet and Tshering $\frac{2}{8}$ of his share from the packet. Compare and find out who has eaten more biscuits?

As presented, the story seems to connect with students’ daily life activities at home and experiences beyond classroom learning of mathematics, except with the concept of $\frac{2}{8}$ of the packet, which is hardly practised in reality. The content is realistic for students and appropriate for the grade level to be understood. In this way, students could easily arrive at the answer demanded by the story. Based on the questions asked, there was provision for the students to communicate and represent their thinking and understanding through different modes such as:

- Enactively by counting the number of biscuits in each packet and comparing the fractional parts, as given in the story
- Iconically by drawing diagrams and comparing the representation of fractions, as illustrated in Figure 7.6.

*Figure 7.6. Fraction strips.*
In this task there was provision for students to be able to make sense of the problem on their own given the opportunity to make connections with their real situation or something they can readily imagine. Hence, in this activity, students could be described using the understanding indicator, *participate in activities in which they identify and describe the developing horizontal and vertical connections within their own conceptual schema*, thereby applying process standards (fourth intention) to develop problem-solving skills.

When analysed, the practice of five participating teachers using the 3+1 framework, under exploration, there was some indication of the curriculum intentions in most of the lessons conducted; for instance, teachers often explained the concept using many strategies. However, the focus was on the teachers’ performance rather than students’ engagement in activities as intended in the new curriculum; thus, there was a mis-alignment between the intention and implementation of the new curriculum.

**Exposition**

According to the philosophy of the new curriculum, the depth of exposition to be conducted is expected to be based on the level of students’ understanding indicated from the performance of the task during the exploration component of the lesson, the Try This activity. In the real classroom situation, observations revealed during exposition that teachers routinely engaged in a range of strategies including:

- Presenting a story problem
- Explanation through closed questioning
- Demonstrations
- Discussions.

Of all the different practices adopted by the participating teachers, the most commonly observed was explanation through closed questioning. This practice was seen in almost every lesson taught by the five participating teachers. An example from Sangay’s third lesson is as follows:

Teacher: How can you create equivalent fractions?

Student 1: By dividing and multiplying….

Teacher: Multiplying and dividing by what?....
Student 2: By multiplying and dividing by numbers…
Teacher: By numbers?
Student 3: Dividing and multiplying by a fraction
Student 4: Dividing and multiplying numerator and denominator……
Teacher: By? By same number?
Student 5: By same number! [Repeated after the teacher]

In Sangay’s questioning, the flow appears smooth and achievable in terms of the teacher’s goals, however, it is not a technique considered a good example of engaging students in active learning because only one student is involved. In this case, students could be described as using the understanding counter indicator, does not describe the vertical and horizontal connections they perceive between mathematical ideas. It represents a model of teaching in which the teacher has the right answer and is seeking confirmation of that answer without allowing for any suggestions of alternative answers. In this case, a teacher can be described as using the understanding counter indicator use of learning activities which do not incorporate opportunities to explicitly explore the connections between mathematical ideas.

A closed questioning session may not engage students in terms of learning taking place, other than recalling previous knowledge: little thinking was encouraged in students to construct their own knowledge. In this way, it was difficult to evaluate deep understanding of the concept by the majority of students in the class, as the teacher tended not to ask any further questions based upon the responses given by students. Instead, they moved on to the next question. Moreover, the response to the question asked was given mostly by a few enthusiastic students. Hence, this type of practice is mis-aligned with the intentions of the new curriculum.

The next commonly observed practice was teacher-led discussions intended to convey the meaning of the mathematical topic. However, the conversation frequently appeared to be shallow and failed to provide opportunities for students to explain their reasoning. This is illustrated by the following exchange observed in Norbu’s third lesson:
Teacher: [Showed the shaded paper of $\frac{2}{4}$ and asked] what is the equivalent fraction of this?

Student 1: $\frac{1}{2}$

Teacher: [Explained the reason for $\frac{1}{2}$ in detail and asked] What is the equivalent fraction of $\frac{1}{2}$?

Student 2: In chorus, $\frac{2}{4}$ [two by four]

Teacher: How did you get $\frac{2}{4}$ two by four? [Before the students could respond, the teacher said], ‘by multiplying...both the denominator and numerator by 2’ [the teacher then showed the procedure on the board]

As evident in the dialogue above, although the activity was designed with the opportunity to justify the answer, the teacher explained the reason rather than seeking ideas from students. In this activity, the teacher could be described using the understanding counter indicator use of learning activities which do not incorporate opportunities to explicitly explore the connections between mathematical ideas. In this way, students were provided with neither a chance to neither think and communicate their ideas, nor represent their understanding in various modes such as enactive, iconic and symbolic. Thus, students could be described using the understanding counter-indicator, passively listen to teacher explanations and does not describe the vertical and horizontal connections they perceive between mathematical ideas.

Gawa was observed in his fifth lesson using a similar style, when students were provided with limited opportunities for thought-provoking processes. He reproduced the following rules on the LCD projector screen and asked the students to read them:

- If the numerators are same, then the fraction with lower denominator is the largest
- If the denominators are same, a fraction with a bigger numerator is the largest.
As indicated above, Gawa presented ready-made rules for comparing fractions, limiting students in communicating their thinking verbally or symbolically. At the same time, he did not provide an opportunity for students to represent their understanding in various modes such as enactively or iconically. In this activity, a teacher’s practice could be described by the reasoning counter-indicator encourage reliance on ready-made rules and formulae without helping students to understand the origin of these rules or formulae. This type of behaviour indicated that teachers still favoured practising the transmission approach of teaching mathematics, and were limited in helping students understand the concept through the process of reasoning and proof.

However, there were some interesting exceptions in the observed lessons, including activities such as presenting a teacher designed problem for discussion with the whole class. The first example of this was from Phurba’s lesson on comparing fractions. In his seventh lesson, Phurba exhibited an unusual teaching example: students were asked to compare Pema and Sonam’s test scores through two identical diagrams, as presented in Figure 7.7.

![Figure 7.7. Phurba's illustrations of test scores.](image)

Students were given the opportunity to justify their answer through an iconic form of representation to depict their understanding more precisely and clearly. Similarly, students could represent the same information symbolically as: \( \frac{3}{5} > \frac{4}{8} \). Also, to compare the given fractions, there was provision for students to communicate their understanding through diagrams and share their reasons.

In this activity, students could be described as using the reasoning indicator, explain their thinking and mathematical activity and flexibly use written and spoken language and represent in symbolic, iconic and enactive forms. Moreover, the task appeared to be authentic: students could easily apply process standards, as the content of the word problems was based on students’ previous knowledge and experiences by using familiar names and test scores. A second example of
exceptional practice was from Sangay, who began her second lesson by pasting a chart paper with drawings of fractions models with three identical rectangles. Each rectangle was divided into five equal parts and shaded to represent $\frac{3}{5}$ as $\frac{3}{5}$, reproduced in Figure 7.8.

![Figure 7.8](image.png)

*Figure 7.8. A model representing fractions in iconic and symbolic modes.*

Students were asked to imagine the three rectangles as three apples to be shared among five friends and then to explain the process of getting each friend’s share of the apples as a fraction. Although students were not provided with physical materials to manipulate, they were at least guided through the process of iconically expressing the meaning of fractions as division. By doing this, Sangay was trying to connect vertically with students’ previous knowledge of shapes and basic concepts of fractions, which aligns with the first curriculum intention (emphasis on relational understanding) using the understanding indicator *connecting vertically between mathematical ideas of varying levels of abstraction or sophistication.*

At the same time, Sangay’s activity also provided an opportunity for connecting horizontally because students were asked to imagine those three rectangles as apples to be shared among five friends. This was Sangay’s attempt at presenting a realistic task which aligned with the third curriculum intention. In addition, Sangay’s presentation of identical diagrams with equal partitions was an attempt to justify why and how $\frac{3}{5}$ in visual form equals $\frac{3}{5}$. This was to help students make sense of the concept of fractions as division. Moreover, an opportunity was seen for students to represent their understanding either through iconic or symbolic form while explaining their answers, thus demonstrating the reasoning indicator *explain their thinking and mathematical activity and flexibly use written and spoken language and represent in either of symbolic, iconic and enactive forms.*
The third exceptional practice was a lesson from Phurba which included a demonstration of how to find an equivalent fraction. Phurba’s demonstration is summarised as follows:

1. Take out the ruler
2. Draw two identical rectangles with 4 cm as length and 2 cm as width
3. Shade the parts according to the teacher’s instructions
4. Divide the 1st rectangle into four equal parts and the second one into eight equal parts.
5. Shade 2 parts in the 1st diagram and 4 parts in the 2nd diagram and discuss the same with the class. Do you see any difference? Similarities between the diagrams?
6. Length – 4 cm but shade in two parts to represent $\frac{2}{4}$
7. Length – 4 cm but shade in four parts to represent $\frac{4}{8}$. Are they same?
8. Yes, equal. So, such a pair of fractions is called equivalent fractions…and [wrote the topic on the board as ‘equivalent fractions’]

Phurba led the students towards the concept of finding equivalent fractions in this step-wise demonstration. Such a guided activity could engage students to be physically active but seemed to be limited in terms of students having to think and identify the connections both horizontally and vertically within their own conceptual schema. In this activity, a teacher can be described as using the understanding counter indicator, use of learning activities, which do not incorporate opportunities to explicitly explore the connections between mathematical ideas. A deeper understanding of the topic was further limited when students were asked to find equivalent fractions of $\frac{2}{4}$ individually in an abstract form. However, from Phurba’s perspective, he was at least trying to introduce the concept more practically and visually for students to make sense of the concepts more quickly by involving students themselves, although not much could be achieved cognitively.

The fourth exceptional example was from Tempa, who initiated another method of introducing equivalent fractions based on shaded portions to convey the concept of equivalent fractions. Tempa’s use of diagrams and its shading to enhance
the understanding of equivalent fractions as presented in Figure 7.9, by which students were exposed to the concept of equivalent fractions visually.

Figure 7.9. Tempa's sample learning activity.

However, students were left without any hands-on activities to explore the knowledge and ideas on their own, except for a few students who were invited to find the equivalent fractions on the board using diagrams. In the process, students were limited to the method introduced by the teacher, depriving them of the opportunity to explore methods themselves. Hence, in this activity, students’ behaviour is aligned to the reasoning counter indicator do not flexibly use a range of language and symbolic/iconic/enactive representations to help students justify the answers.

The overall findings regarding the exposition component of the lesson, when analysed using the 3+1 framework, could be described mainly by counter indicators in relation to all of the curriculum intentions associated with both students and teachers. Nevertheless teachers appeared to be at least aware of the curriculum intentions in terms of using examples connected with students’ daily activities. At the same time, using different strategies to help them explain the concepts meant the lesson was mostly dominated by the teacher. Such practices from the teachers’ side left little opportunity for students to become engaged in activities and construct their own knowledge.

Formative Assessment

According to the new curriculum, formative assessment strategies are suggested to link all the lesson components. For instance, the content of the activity assigned for exploration is expected to connect the current lesson topic with previous lessons and intended future lessons. However, the majority of the teachers
demonstrated practices in conducting formative assessment which were inconsistent with those suggested by the new curriculum, such as linking the Try This activity B to the Try This activity A used in the beginning of the lesson during the exploration.

In regard to formative assessment, the most popular approach adopted by participating teachers was to invite students to share their work with the whole class on the board. For instance, in one of the Phurba’s lessons, having conducted an exposition on finding equivalent fractions using the fraction model, he provided a similar learning activity on representing equivalent fractions using the pair of fractions, $\frac{1}{3}$ and $\frac{2}{6}$, presented in a diagram as shown in Figure 7.10. This type of assessment does not qualify to be addressed as formative assessment in terms of overall understanding of the concept. It is, rather, a fixed activity to reproduce similar activities conducted by the teacher. Thus, there is a doubt in terms of teachers’ knowledge of the conduct of authentic formative assessment.

![Diagram showing 1/3 and 2/6 shaded]

Figure 7.10. Phurba's sample learning activity.

The other participating teachers had similar methods of conducting formative assessment. In this way, the participation of students in formative assessment was limited. The benefits were only for those presenting the work: the remaining students were left inactive. In such activities the students could be described as using the reasoning counter indicator, *passively listen to teacher explanations.* This is concerning, given the purpose of formative assessment to assess what has been learnt.

Based on the performance of those few chosen students, teachers either moved on to the next activity or repeated the explanation again to the whole class. In this way, there was little opportunity for a teacher in understanding the depth of knowledge learnt by the majority of students before moving on to the next topic. For
instance, in one of Phurba’s lessons, the video camera captured a student who had drawn diagrams to represent fractions as illustrated in Figure 7.11.

Unfortunately, at no time during the lesson did the teacher notice that the student had made a mistake. As a result, this student was left with a misconception of fractions. More students may have had similar misunderstandings, but the teacher did not devote any time to evaluate the quality of the students’ learning. Another common practice in formative assessment was assigning a short duration activity, as reproduced in Figure 7.12.

There was no indication of the ‘why’ part in the given tasks and thus they did not cognitively challenge Class 5 students. They contained only the ‘what’ part of the question and therefore students could be described as using the reasoning counter indicator, focus on achieving correct answers without explaining or justifying the processes used. This indicated Phurba seemed unable to implement the importance of curriculum intention on reasoning.

Although most of the participating teachers practised formative assessment by inviting students to the front to present to the class without much classroom engagement, there were a small number of examples that were quite thought provoking. Norbu, in her second lesson, assigned students a challenging yet accessible task, allowing multiple method of solution. The task was: use any two
methods to represent $\frac{5}{2}$. Asking students to express $\frac{5}{2}$ in any two methods could encourage students to reason and to communicate either orally or in written words. Moreover, such an activity is provocative enough for students to justify their understanding in various modes, such as:

- **Enactively**: Sharing practically five identical chocolate bars equally among two friends, each receiving two bars and a half.

- **Iconically**: Drawing five identical shape diagrams breaking each into two parts to be shared among two friends (Karma and Dorji) and each receiving five halves or two whole shaded and one half shape shaded, illustrated as follows.

\[
\begin{align*}
\text{Karma} & \rightarrow \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \text{ (symbolic)} = 5 \text{ halves or } 2 \text{ whole and a half (expressed in written words).} \\
\text{Dorji} & \rightarrow \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \text{ (symbolic)} = 5 \text{ halves or } 2 \text{ whole and a half (expressed in words)}
\end{align*}
\]

Symbolically $\frac{5}{2} = 5 \div 2 = 2 \frac{1}{2}$.

As presented above, students able to communicate their thinking would most likely increase the depth of their understanding of the given concept. Moreover, providing freedom for students to choose any method to express their understanding adds more value to the given problem and thus, in this activity, students can be described as using the understanding indicator *participate in activities in which they identify and describe the developing horizontal and vertical connections within their own conceptual schema.*

The findings from this lesson component indicate that the use of a formal practice of formative assessment was very rare in the 28 lessons. Moreover, there was no sign of a teacher using any kind of formative tools other than inviting a few selected students to the board to demonstrate to the rest of the class at the end of the activity being performed. This clearly indicated that there is high chance for teachers not being able to differentiate between formative assessments from any other routine
practice of assigning students (i.e., with some computational activities) towards the end of the lessons. However, since the students were only occasionally asked to participate in thought-provoking activities related to key concepts, effective formative assessment procedures were rarely observed.

Practising and applying

The fourth lesson component suggested by the new curriculum was practising and applying. This component is intended to provide students with extra activities to enhance their understanding of the concepts learnt in the classroom. To help teachers, the new Guidebook included a page or two of suggested activities for every topic to supplement activities prepared by individual teachers. However, not all the participating teachers attempted to use these suggested learning activities, but two teachers designed their own activities. For instance, in her second lesson, Sangay attempted to use as an example of sharing five watermelons among six friends to help students apply the concept of fractions as division, as reproduced in the following transcript:

Teacher: Imagine there are five watermelons to be divided among 6 boys. What part of a watermelon does each student get? Let us take one watermelon [into how many parts should I divide it?]

Student 1: 6 parts

Teacher: If I give one part to each boy here, what part of the watermelon will each get?

Student 2: One by sixth of watermelon

Teacher: 2nd watermelon…what part will each boy get?

Student 3: two by sixth

Teacher: 3rd, 4th, 5th, sixth?

Student 4: Three by sixth, 5 by sixth

As transcribed above, the learning task was discussed in the class through a closed questioning session by selecting a watermelon as the object of the lesson and how one might share five melons among six boys. In this activity, the teacher can be described as using the context indicator, *base learning activities on students’ reality.*
Such a problem can also be constructed in a thought-provoking way as posed in the following scenario:

Six boys were invited for a party to celebrate one of their birthdays. It was a hot day and the mother of the boy asked the guests to bring a melon but she did not have time to buy one herself. So when the party started each of the five visiting boys brought a melon. So the problem she now faced was to work out how to share the five melons among the guests and her son.

However, such practices were found infrequently. Instead, teachers tended to choose the routine calculation problems rather than providing a thought-provoking task.

**Summary**

When all of the lessons were analysed in terms of curriculum intentions using the 3+1 framework and the four lesson components structure, most practices did not align with what was being suggested by the new curriculum. Among the four components of the lesson, the second component of exposition was most frequently observed but conducted mostly through a closed questioning approach. Moreover, as described in Section 4.5.2, the teacher was for most of the time positioned in front of the class, consistent with a teacher-centred approach.

The least evident intention was the emphasis on reasoning: students were provided with minimal opportunity to justify their answers. The use of context was quite common but mostly as simple examples to help teachers explain the concept, and not as contextualised complex tasks for students to engage with as intended by the curriculum. In most of the lessons, instead of contextualised activities provided for students to solve on their own, participating teachers generally used context related examples while trying to explain the targeted mathematical concept.

Despite several challenges faced while implementing the new curriculum, the five participating teachers presented some interesting activities in their lessons that did align with aspects of the curriculum intentions. For instance, a thought-provoking activity in the beginning of Gawa’s second lesson was a positive indication of teachers’ potential to conduct an effective lesson. It was effective in terms of encouraging students to think and participate in solving the problem. In the process of thinking, students were encouraged to identify and develop connections both vertically and horizontally to shape the problem. However, there were few lessons in which thought provoking activities were used. Such evidence indicates that the
teaching was lacking in terms of emphasising students’ justification of their understanding, indicating teachers’ transfer of knowledge rather than transformation of knowledge, and supporting instrumental beliefs (McLeod, 1992).

However, towards the end of the unit, improvement was seen with some of the participating teachers, particularly Phurba, who started to use more activities aligned with the intentions of the new mathematics curriculum. For instance, he began to reduce the time students spent listening passively to his explanations and instead engaged them in activities through which they could explore and construct their own knowledge. Moreover, he attempted to design learning activities that would integrate both horizontal and vertical connections for students to be able to explore using appropriate process standards.

This progress was in contrast to his opinion shared during a post-lesson interview in which he indicated he did not favour the students’ involvement in doing something on their own in the class, justifying this with regard to covering the syllabus in time. As a general observation, there is hardly any exploration component implemented in the majority of lessons conducted. Hence, it appears that of all the components, exposition dominated the lessons.

7.2.3 Past examination papers
To help provide a deeper understanding of the practice of teachers in terms of implementation of the new curriculum, the researcher examined some of the past examination papers (e.g. 2012) for the chosen class level. When analysed, in one of the annual examination papers from one of the schools, it was found that most of the questions required mere recall of factual knowledge. There were very few thought provoking questions requiring students to apply their knowledge through deep thinking. In this sense, most of the knowledge required in answering the questions was of the procedural-type, and very little conceptual-type knowledge was demanded of the students (see e.g., the questions in Figure 7.13.)
Likewise, the pattern was repeated in the questions asked during the unit test conducted by the participating teachers in the two schools. A unit test was conducted at the end of the unit on fractions at both schools using the same paper containing five questions, of which all required procedural knowledge in answering. The test is shown in Figure 7.14.
1. If 5 boys share 3 large biscuits equally, what fraction of a biscuit will each boy get? [2 marks]

2. Write $\frac{37}{4}$ as a mixed number. [1 mark]

3. Write two equivalent fractions for each. [2 marks]
   a) $\frac{5}{6}$
   b) $\frac{3}{4}$

4. Which fraction is greater? [2 marks]
   a) $\frac{7}{9}$ or $\frac{3}{9}$
   b) $\frac{32}{4}$ or $\frac{18}{5}$

5. Add and subtract using fraction strips. [3 marks]
   a) $\frac{3}{10} + \frac{4}{10}$
   b) $\frac{3}{5} - \frac{1}{5}$

Figure 7.14. Sample questions from the unit test (participating teachers).

7.3 CHAPTER SUMMARY

The overall approach adopted by the five participating teachers was teacher-centred. For instance, in terms of using the lesson plan format, all participating teachers seem to have followed the same format reproduced in Figure 7.1. This contrasts to the format suggested by the new curriculum which was presented in Chapter 2. The format of lesson plans adopted by participating teachers was composed of four main phases. This approach was characterised by strategies where teachers would explain mathematical concepts in detail and students were left listening passively. Moreover, Norbu (during an interview) stated that teachers are forced to follow the traditional approach as students are not used to the modern approach as intended in the new curriculum.

Limited activities were provided in which students could identify and describe, developing horizontal and vertical connections within their own conceptual schema, although most of the participating teachers appeared able to impart knowledge. However, in most of the lessons, participating teachers paid more attention to delivering explanation of mathematical concepts. Hence, the participating teachers did not seem to practise approaches that engage students in the type of learning experiences consistent with constructivist assumptions about learning.
Through the analysis of the lessons, application of context was found to be quite popular, although not in an authentic form. Having demonstrated several practices of delivering a context-related example, participating teachers appeared to be aware of the importance of context-based experiences in conducting a lesson. However, observations indicated that teacher participants were limited in their practice, despite their awareness of theory. The findings here indicate that the common practice in terms of planning, conducting lessons and information gathered from informal post-lesson interviews provide evidence that teachers possess Platonist beliefs more so than experimental beliefs about mathematics and mathematics education, thus addressing Research Questions 1, as shown in Figure 7.15.

![Figure 7.15. Findings for Research Question 1.](image)

The participating teachers appeared to be aware of curriculum intentions but they did not deliver lessons that aligned with them, particularly in terms of engaging students in constructing their own knowledge and mathematical ideas as intended in the new curriculum. Hence, their practices tended to reveal more of a traditional teacher-centred chalk and talk method, in this way, addressing Research Question 2 as in Figure 7.16.

![Figure 7.16. Findings for Research Question 2.](image)

Since, teachers’ conduct of lessons was teacher-centred, and thus did not align with the curriculum intentions. There was a mis-alignment of teachers’ practice with curriculum intentions, as indicated in Figure 7.17.
The findings from both the phases of the study were presented in Chapters 5 and 7, and discussed in Chapter 8 in terms of each of the four Research Questions.

Figure 7.17. Findings for Research Question 3.
This study set out to explore the teaching of mathematics in Bhutanese primary schools subsequent to the introduction of a reformed mathematics curriculum based on NCTM standards and incorporating broad pedagogical strategies informed by social constructivist learning theories. The goals of the new curriculum were expressed in terms of four intentions. Three of these related to pedagogical practices and the fourth reflected the processes in which students should engage to achieve deep understanding of mathematics at the primary level. The objectives of the study were expressed as four Research Questions:

Question 1: What are the beliefs of Bhutanese primary teachers about mathematics teaching?

Question 2: What are Bhutanese primary teachers’ planning and classroom practices in teaching mathematics?

Question 3: To what extent are mathematics teaching practices aligned with curriculum intentions?

Question 4: What influences the implementation of the curriculum intentions?

The case study involved a macro-level phase in which teacher beliefs and their approaches to designing learning activities were explored through a survey of 80 teachers from 40 randomly selected schools across Bhutan. These data were reported in Chapter 5. The inconclusive findings of this phase, in terms of alignment between intention and implementation of the new curriculum, indicated that teachers’ expressed beliefs supporting curriculum intentions but in practice it was the other way around. Consequently, the micro-level phase was conducted to explore further; five primary teachers were observed teaching mathematics in two schools, one urban and one rural. These data were reported in Chapters 6 and 7. An analytical framework to explore these questions was presented in Chapter 3 and was referred to as the 3+1 framework, indicating the three teaching aligned intentions and the intention for student mathematical activity.
In this chapter, the discussion which responds to the four research questions addressed in five sections: The first section explores Research Question 1, drawing primarily on findings of the beliefs survey which was supplemented with interview, planning and classroom observation. The second section addresses Research Question 2. In this section, teaching practices and their interpretation in relation to curriculum intentions and contemporary teaching literature are presented. In the third section, Research Question 3 regarding the alignment of teaching practice with the curriculum intentions, is examined. The fourth section presents a synthesis of the findings from all the data and discusses these findings relevant to Research Question 4, namely issues that constrain or support teachers in their adoption of the reformed curriculum. The fifth and final section summarises the discussions.

8.1 BELIEFS ABOUT TEACHING MATHEMATICS

In this section, Research Question 1 is addressed: What are the beliefs of Bhutanese primary teachers about mathematics teaching? Based on the questions posed in the macro-level survey, respondents’ beliefs were explored in three categories, namely:

- Beliefs about the nature of mathematics (n = 6)
- Beliefs about the learning of mathematics (n = 7)
- Beliefs about the teaching of mathematics (n = 8)

Each dimension had a set of belief statements requesting responses on a five-point Likert scale. The belief statements included some aligned with and others not aligned with the curriculum intentions which are associated with a constructivist approach of teaching and learning of mathematics. For instance, the first dimension listed above sought beliefs about the nature of mathematics. Thus, the respondents agreed or disagreed with certain statements based on their beliefs about the nature of mathematics. Basically, how mathematics appears depends on the type of exposure and experiences associated with the subject, and beliefs statements depicting how mathematics can be learnt and taught that matched with their existing or entrenched beliefs.

As discussed earlier, the survey was completed by primary school teachers teaching mathematics from 40 randomly selected schools across the country. Three sets of questionnaires were distributed to each of the 40 primary schools making the
total of 120 sets. The three sets of questionnaires were addressed to school principals, who in turn distributed them to any three teachers teaching mathematics in the school. Respondents were requested to post back completed questionnaires to the researcher. Of the 120 distributed questionnaires, 80 were returned and analysed to reveal the key findings as shown in Table 8.1. Overall the teachers’ responses suggested beliefs about teaching mathematics that were consistent with social constructivist learning theories, as discussed in Section 3.2.

Table 8.1

<table>
<thead>
<tr>
<th>Belief categories</th>
<th>Key Findings</th>
<th>Type of Beliefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature of mathematics</td>
<td>The majority of respondents tended to support the statement describing the nature of mathematics aligning with experimental beliefs (Ernest, 1989)</td>
<td>Experimentalist</td>
</tr>
<tr>
<td>Learning of mathematics</td>
<td>The majority of the respondents agreed strongly with the idea of encouraging children in mathematical thinking processes and that they are capable of high levels of thought (Goldsmith and Mark (1999)</td>
<td>Experimentalist</td>
</tr>
<tr>
<td>Teaching of mathematics</td>
<td>The majority of the respondents believed it was important for teachers to design instructional approaches that were well-organised and which would facilitate students’ construction of knowledge, supporting experimentalist beliefs.</td>
<td>Experimentalist</td>
</tr>
</tbody>
</table>

As Leatham (2006) pointed out, beliefs cannot be perceived or measured but must be inferred from what people say, intend and do. It was indicated in Table 8.1 that, based on the beliefs statements describing on each of the belief categories, the majority of the respondents identified as holding experimentalist beliefs. Results from the questionnaires appeared to indicate that, generally, teachers held beliefs aligning with social constructivist approaches to teaching and learning mathematics. Given these findings, it would be reasonable to expect that teachers would plan and implement practices aligned with constructivist principles. Such principles would be consistent with the literature discussed in Chapter 3, Section 3.2.

Although the majority of the respondents expressed experimentalist beliefs, in practice, they manifested Platonist beliefs, as discussed further in the following section.

8.2 **PLANNING AND CLASSROOM PRACTICES**

Winston Churchill, the former Prime Minister of the United Kingdom stated that “plans are of little importance, but planning is essential”. Superfine (2008) argued
that “planning commonly refers to the time teachers spend preparing and designing activities for students” (p. 11). The assumption is that teachers’ plans reflect their beliefs. The second research question addresses participating teachers’ planning for and practices in lessons: What are Bhutanese primary teachers’ planning and classroom practices in teaching mathematics? This section comprises three sub-parts. Firstly (Section 8.2.1), teachers’ planning as revealed in the macro-level survey is summarised, in the second part (Section 8.2.2) lesson plans are discussed, and in the third part (Section 8.2.3) the micro-level analysis of the observations are discussed.

8.2.1 Teachers’ learning activity planning

This section discusses the findings emerging from the final section of the survey, in which teachers were asked to design a sample learning activity. As discussed in Chapter 5, the collection of sample learning activities on teaching fractions was analysed to explore the relationship between respondents’ beliefs and practices about mathematics and mathematics teaching. The findings indicated that there is a gap between respondents’ expressed beliefs and their design of sample learning activities. Although the majority of respondents were able to demonstrate a strong reform-oriented viewpoint in their beliefs, they showed limited evidence of implementing a pedagogy that matched those beliefs.

Analysis of all respondents’ sample learning activities indicated that the majority of respondents planned predominantly procedural/mechanical approaches rather than experiences likely to foster genuine understanding. These findings are consistent with other studies in Bhutan conducted by the Royal Education Council & Educational Initiatives (2010). Likewise, a study conducted in the USA by Cooney (2001) argued that interest in teachers’ beliefs has been grounded on the conjecture that what teachers do in their classrooms is a product of their beliefs. This seemingly contradicts the findings of the present study in regards to beliefs expressed in the survey questionnaires and teachers’ sample learning activities. Hence, it appears that there is a gap between teachers’ expressed beliefs in regards to the intentions of the curriculum, and their implementation of it. This finding is similar to that of Perrin (2008), who claimed that there was no significant correlation found between teachers’ beliefs scores and their reported level of the use of NCTM-aligned practices.
Research by Keys (2007) has shown that teachers possess three categories of beliefs which he described as expressed beliefs, entrenched beliefs and manifested beliefs. Accordingly, all three categories of teachers’ beliefs (expressed, entrenched and manifested) were revealed in this study to a certain degree. For instance, according to Keys (2005), expressed beliefs were defined as “expressed verbally and are at times consciously acted upon” (p. 505). As such, in this study, a result derived from survey responses tends to fit with Key’s definition of expressed beliefs, indicated by the respondents in the survey questionnaires. However, the type of beliefs manifested in the sample learning activities designed by survey respondents did not align with the way they responded to the belief statements. Hence, in this study, analysis and comparison of expressed and manifested beliefs reveal some inconsistencies.

Further, when explored by analysing the observed lessons of the five participating teachers, the observed lessons tended to align with the beliefs indicated by the sample learning activities compared to expressed beliefs as inferred from their responses to the Likert type items in questionnaires. In this case, the manifested beliefs generated from the way lessons were planned and conducted appeared to support beliefs generated from the design of sample learning activities: outworking of the Platonist beliefs. Hence, in terms of practice in the form of planning and conducting lessons, the findings did not match with expressed beliefs displayed when responding to belief statements. Thus, in this study, two different types of teachers’ beliefs (expressed and manifested) were identified that did not match.

In the macro-level study, an apparent gap existed between the expressed beliefs and approaches that teachers indicated in their planning of a mathematics activity. This planning task however, only provided a snapshot of teachers’ intentions. Stigler and Hiebert (1999) have used the metaphor of teaching as a machine with the parts operating together and reinforcing one another, driving the vehicle forward. Lesson planning is only one part of the machine of teaching. A more comprehensive insight into the practices of teachers was necessary to understand how planning and implementation were aligned with beliefs, as argued by Roche, Clarke, and Clarke (2014) that “what teachers choose to commit to writing provides a lens into what they possibly hold to be most important” (p. 856).
Fernandez and Cannon (2005) distinguished planning from preparation. According to these authors, planning refers to activities related to knowing what to teach and how, and the preparation of activities associated with obtaining or designing materials and setting up the classroom and work spaces. Further, as argued by Roche et al. (2014), mathematics teaching involves the ongoing and interactive adaptation of planning. It is essential for teachers to be aware of all aspects of planning at all stages. The planning data provided through the survey were not conclusive and required a more intense examination of practices via the actual planning and conduct of lessons in the micro level phase. Classroom practices of five selected teachers were explored, drawing on three different sets of data:

- Collection of plans for 28 lessons
- Classroom observations of 28 lessons
- Post-lesson interviews.

8.2.2 Teachers’ lesson planning

As discussed in Chapter 7, plans for every lesson taught during the observation period of the micro-level phase were collected by the researcher to further explore the nature of lessons to be conducted. Superfine (2008) argued that it is very important for teachers to pay attention in planning their lessons. According to Kilpatrick, Swafford, and Findell (2001), planning of a lesson is considered a core routine of teaching. As argued by Moyer and Milewicz (2002), teachers must plan the type of questions to be asked during the lesson, which could monitor students’ thinking about the target concept and also inspire them to explain their thoughts and ideas without giving them too much information. For all these reasons, lesson plans from each participating teacher were summarised and analysed and it was found that all five teachers used a similar format. The format used was something teachers had been exposed to during their training period. This format could be characterised as a lesson with a fixed, routine procedure of information input based on the list of lesson objectives. Similarly, the learning activities depended on the type of information delivered and a lesson is expected to end with a brief lesson closure activity to evaluate the learning based on the list of objectives planned.

This type of approach does not seem to align with the social constructivist approach intended in the new curriculum. Rather, the detailed lesson planning
provided by teachers in the micro-level phase was consistent with the design of sample learning activities provided in the macro level phase. As previously argued, planning is only one part of the machine of teaching. Although, their lessons plans may not provide a reliable indicator of actual practice, they can be used as a useful lens for understanding the relationship between teachers’ experiences, ideas of mathematics teaching and learning, and the alignment of the curriculum intentions in planning development (Superfine, 2008). Hence, the decision to explore the actual teaching practices of a selected group of teachers through observations was made.

### 8.2.3 Classroom practice

In this section, the practices of five primary school teachers are discussed. The analysis of classroom practice was carried out based on observation and the video recorded lessons taught on Fractions in Class 5 by the five participating teachers. A total of 28 observed lessons were analysed using the 3+1 framework, through the four lesson components suggested by the new curriculum. When lessons were coded and analysed in terms of the four lesson components, the most dominant component was exposition. The results indicated that the majority of participating teachers were good explainers, exhibiting Platonist beliefs as discussed in Section 7.1. In almost all cases the relevant mathematical concept was explained by the teacher, leaving little opportunity for students to actively discuss, explore and construct their own understanding.

Although all participating teachers appeared to believe in a constructivist approach to teaching and learning of mathematics as indicated in the Likert type items from the surveys, they did little to engage students in thought-provoking activities. Most of the time, as indicated in their lesson plans, teachers were busy explaining, demonstrating and solving problems for students. This finding is quite similar to the one observed in the study, the role of teachers’ beliefs and knowledge in the adoption of a reform-based curriculum in USA by Roehrig and Kruse (2005). These researchers found that, although the curriculum was designed for the class to spend equal amounts of time on whole class, small group, and individual work, teachers spent 70% of their time giving directions and lecturing.

Aligning with the findings discussed in the preceding paragraph, teachers in this study did not practise what was intended in the new curriculum. Although findings from the survey questionnaires indicated that the majority of the respondents
supported belief statements aligned with the intention of the new curriculum, they failed to design learning activities accordingly. In terms of designing sample learning activities, the majority of the respondents exhibited Platonist and not experimental beliefs as expressed while responding to the belief statements. Such results tend to contradict to some extent findings of past studies in terms of alignment between teachers’ beliefs and practices.

Roehrig and Kruse (2005) conducted a study on the role of teachers’ beliefs and knowledge in the adoption of a reformed-based curriculum in a school in California. The main purpose of their study was to understand the impact of a reformed-based chemistry curriculum; they found that teachers’ beliefs shape classroom practices. Their findings were consistent with those of earlier researchers such as Guskey (1986), Richardson (1996), and Tobin and McRobbie (1996), who claimed that teachers’ beliefs have a significant impact on their classroom practices, and that teachers are able to implement practices consistent with their beliefs about teaching and learning approaches. However, in this study, the overall findings are not consistent with those of past researchers in terms of beliefs and practices. In this study, a mis-match was found between teachers’ expressed and manifested beliefs. The methods and practices adopted by the teachers when conducting lessons were not aligned to beliefs as expressed in the survey questionnaire. Thus, there is an indication of mis-alignment between intention and implementation of the new curriculum, as discussed further in the following section.

8.3 ALIGNMENT OF INTENTIONS AND PRACTICE

This section discusses Research Question Three: To what extent are mathematics teaching practices aligned with curriculum intentions? Having analysed the results from both macro and micro level phases of this study, there appeared to be a mismatch in terms of teachers’ intentions and their implementation of the curriculum. Teachers in the current study showed a lack of understanding of the actual practice of teaching and learning of mathematics in ways aligned with the philosophical intentions of the new curriculum. They demonstrated few lessons that created a collective learning environment for students to construct ideas and modify them through interactions. Their practices appeared to be in consistent with Tarmizi et al. (2010), who argued that the teachers’ task is to create a particular kind of collective environment for learning.
According to the findings from the macro-level phase, the majority of respondents had beliefs aligned with the intentions of the new curriculum. Their responses to the statements describing the new curriculum showed acceptance of the ideas based upon social constructivism, as presented in Table 5.17. However, when observed in-depth, a gap was evident between the intention and implementation of the new mathematics curriculum. Of the six statements included under the fourth category (new curriculum) in the survey, the most prominent conflict was observed with the response given to the following statement: the idea of providing learning activities at the beginning of every lesson (e.g., Try This section) is a very enriching idea for the student). In terms of mean and median, the response given for the above statements indicates very strong support from the survey respondents. However, in terms of practice, the concept of conducting the Try This activity with these five participating teachers was one of the neglected components in most lessons. This was also seen in the analysis of the 72 sample learning activities (macro-level phase) and 28 lesson plans followed by lesson observations (micro-level phase). There was an indication of limited understanding of the curriculum intentions reflected in all these artefacts.

The sample learning activities (from the macro-phase study) showed a few activities, which were designed in a similar nature to the Try This activities were in the form of a word problem integrating all curriculum intentions. Such results tend to indicate that the five participating teachers represent the practice of the general population of teachers teaching mathematics in Bhutanese primary schools, and depict mis-alignment between curriculum intentions and the way it is being implemented. This finding is consistent with those from other studies which reveal a mis-match between an intended and an implemented curriculum reform (Cuban, 1993b; Handal, 2001).

Seventy-two sample learning activities were analysed using the 3+1 analytical framework, and it was found that there were few activities where all the curriculum intentions were strongly integrated. In this way, the intentions of the new curriculum were not being achieved. Similarly, when the researcher analysed the plans of the 28 lessons taught by the five teachers in the micro level phase, the findings confirmed what was found in the macro-phase level analysis of the sample learning activities.
Further, all five participating teachers followed the same kind of lesson plan format comprising the four main components, as described in Chapter 7.

These observations suggest a behaviourist rather than a constructivist understanding of learning. Lessons were conducted based on the plans with few alterations and little scope to respond to students’ needs. Consequently, when the 28 lessons were observed and coded based on the lesson format suggested by the new curriculum, the majority of the lessons were dominated by exposition strategies (information input) and other components such as exploration were far less evident. A similar study that explored mathematics teachers’ beliefs and curriculum reform was conducted by Handal and Herrington (2003). These authors’ focus was mainly on the role of mathematics teachers’ beliefs and their impact on curriculum reform. Their study strongly supported the argument that “teachers’ beliefs about the teaching and learning mathematics are critical in determining the pace of curriculum reform” (Handal & Herrington. 2003, p. 59). They concluded that teachers with behaviourist beliefs constrained the successful achievement of constructivist-oriented curriculum reform.

As described in Chapter 5, all sample learning activities were analysed using the 3+1 analytical framework, three important findings were revealed:

- lacked an emphasis on engaging students actively in the given activity;
- designed sample learning activities emphasising ‘what’, rather than ‘why’; and
- provided limited opportunities for students to apply integrated process standards (verbal communication, reasoning, making horizontal connections and enactive representation).

These findings confirm previous reviews conducted within Bhutan. For example, the report provided by the Bhutanese Educational Initiatives and Royal Education Council Educational Initiatives and Royal Educational Council (2010) claimed that the majority of teachers leaned more towards procedural/mechanical teaching than teaching for genuine understanding. Similar findings have been reported in the research literature such as that of An et al. (2011), who argued that the practices of assigning the same problem to every student, teachers’ detailed explanation and limiting students to only one way of solving a problem, are rife and
contribute to students failing to achieve a deep conceptual understanding. Research has shown that to change a teaching style is difficult, because a decision to change one’s practices demands a process of unlearning and learning again (Mousley, 1990). Moreover, teachers find teaching mathematics through traditional approaches easier than attempting reformed methods, which are normally considered burdensome, despite evidence of their advantages (Handal and Herrington (2003). According to Beswick (2006), it is not enough to provide teachers with resources, curriculum materials and ideas without attending to their beliefs.

Despite the fact that the participating teachers showed some awareness of the importance of leading students towards a deeper understanding of mathematical concepts, they were not able to practise the strategy in the classroom. Relatively few students seemed to be prepared to reason and justify their ideas and processes. In most lessons, the students’ main role was to sit passively and listen to the teacher’s explanation. Frequently, students were engaged in activities in which little thinking was required, such as answering closed questions with one-word answers and solving problems mechanically. Consequently, students were given little opportunity to understand mathematical concepts more deeply in terms of connection both vertically and horizontally. As discussed in Section 7.1, the type of practice observed from the five participating teachers tends to align with Platonist beliefs about mathematics, where teachers play the main role in explaining the concept, thereby contradicting the intentions of the new curriculum. Such teachers tend to hold authoritative views about their knowledge and see themselves as experts. They see their role as mainly transmitting their knowledge to their students.

According to the new curriculum, teachers should discard their traditional roles to become facilitators of learning, helping students to construct their own ideas. For instance, the Bhutanese new curriculum encourages teachers to spend equal amounts of time on whole class, small group, and individual work; however, from the researcher’s observation, participating teachers were found to spend more than 75% of their time explaining concepts by using several different types of examples.

Notwithstanding their awareness of the importance of using meaningful contexts, in this study, the majority of participating teachers provided few problems that were contextually meaningful for students to solve either individually or collaboratively. However, the five participating teachers often used context-related
examples to help them to explain a mathematical concept. For instance, as discussed in section 7.2.1, Norbu used two red and one blue lunch boxes to help her explain the concept of thirds, as two-thirds (2/3) of the lunch boxes were red and one-third blue (1/3). These results suggest that participating teachers were aware of ideas associated with the curriculum intentions regarding the context of daily life, but were limited in terms of their application in reality, facing challenges in materialising the ideas effectively to help students learn mathematics.

Teachers’ intended practices were inferred from their planned learning activities, lesson plans and lesson observations of the five participating teachers. The method adopted for this comparison was through five belief items sorted from survey questionnaires, as presented in Table 8.2. The overall comparison of teachers’ expressed beliefs and practices (manifested beliefs) from both macro and micro-level phases. These five items were identified based on common characteristics seen in the lesson observations, which are matched with some of the belief items listed in the survey questionnaires. In this way, it was easier to compare and match teachers’ beliefs (from survey statements) with their intended practice manifested (sample learning activities and lesson plans) and their actual practices (lesson observations).

The overall observation in terms of teachers’ beliefs and practices indicated in the macro-level phase is supported by the findings from the micro-level phase. For instance, in the macro-level, there appeared to be a mis-match between teachers’ expressed and manifested beliefs. As discussed in section 3.1.2, on three different types of teachers’ beliefs by Keys (2005), the majority of teachers expressed beliefs in support of experimentalist approaches thus aligning with the curriculum intentions. However, the same group of teachers manifested Platonist beliefs when designing a sample learning activity which did not align with curriculum intentions. This mis-alignment was further evidenced by the findings from the micro-level phase in lesson plans and observation, where the results indicated practices consistent with Platonist beliefs. Almost all the participating teachers fell back on explanation using all means of teaching aids such as children themselves, pictures/diagrams and examples based on students’ daily life experiences. However, none showed their expertise in engaging students in doing things on their own. Hence, students were deprived of opportunities in using process standards to help them understand deeply and meaningfully.
### Table 8.2

**Comparison of Teachers' Beliefs and their Practices**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Beliefs</th>
<th>Macro Phase Survey</th>
<th>Sample Learning Activities</th>
<th>Lesson Plan</th>
<th>Micro Phase Lesson Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. Mathematics learning is enhanced by activities which build upon and respect student’s experiences.</td>
<td>Experimentalist</td>
<td>Indicated (majority)</td>
<td>Not indicated (majority)</td>
<td>Not indicated (majority)</td>
<td>Not indicated (majority)</td>
</tr>
<tr>
<td>13. Children construct their own mathematical knowledge.</td>
<td>Experimentalist</td>
<td>Indicated (majority)</td>
<td>Not indicated (majority)</td>
<td>Not indicated (majority)</td>
<td>Not indicated (majority)</td>
</tr>
<tr>
<td>14. Teachers should provide instructional activities which result in problematic situations for learners.</td>
<td>Experimentalist</td>
<td>Indicated (majority)</td>
<td>Not indicated (majority)</td>
<td>Not indicated (majority)</td>
<td>Not indicated (majority)</td>
</tr>
<tr>
<td>15. The role of mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge.</td>
<td>Platonist</td>
<td>Indicated (majority)</td>
<td>Indicated (majority)</td>
<td>Indicated (majority)</td>
<td>Indicated (majority)</td>
</tr>
<tr>
<td>21. Mathematics instruction should be organised to facilitate children’s construction of knowledge.</td>
<td>Experimentalist</td>
<td>Indicated (majority)</td>
<td>Not indicated (majority)</td>
<td>Not indicated (majority)</td>
<td>Not indicated (majority)</td>
</tr>
</tbody>
</table>
As indicated in Table 8.2, five of the 21 belief items included in the survey questionnaires were used to compare the link between teachers’ expressed beliefs and their practices (i.e., manifested beliefs). For instance, analysis of the statement *mathematics learning is enhanced by activities which build upon and respect students’ experiences* showed that the majority of respondents agreed or strongly agreed, supporting experimental beliefs about mathematics and mathematics education. However, when the same findings were compared with the teachers’ planning and their actual classroom practices, there was a clear mis-match. Another example regarded the beliefs statement *mathematics skills should be taught in relation to understanding and problem solving*, which received very strong support from the respondents (Mean = 4.46), with Likert scale 5 being the highest and the strongest. However, in practice, few activities were conducted where students were given the chance to help them understand and solve problems more efficiently. A similar pattern was found with the rest of the key belief items, except with the fourth statement (the role of mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge), which appears to contradict the other four belief statements. Unlike the other relationships, where there was a mis-match in findings between expressed and manifested beliefs, findings for this statement appeared to match expressed and manifested beliefs. The following section presents some of the constraints affecting the implementation of the new curriculum.

8.4 IMPLEMENTATION OF CURRICULUM INTENTIONS

Finally, in this section, Research Question 4 is addressed: What influenced implementation of the curriculum intentions? Cheng and Wang (2012) conducted a study on the factors affecting the implementation of curriculum reform in Hong Kong, their aim being to examine key hindering and facilitating factors in schools during the first stage of curriculum reform (2001–2006). Their findings indicated that, consistent with the current literature, recent curriculum reform in Hong Kong faced several key obstacles and challenges common in many other countries. In the implementation stage, the prevailing obstacles were identified as teachers’ heavy workloads, students’ diversity in class, and teachers’ inadequate understanding of the reform. In addition, according to Keys (2003), the faithful implementation of curriculum depends on the inter-relatedness between teachers’ knowledge and their beliefs about the subject that they teach.
Moreover, as discussed in Chapter 3, the fidelity of implementation of reformed curriculum tend to depend on several identified criteria such as adherence, duration, quality of delivery, participant responsiveness and program differentiation. Of these five criteria, the most frequent criteria revealed from this study are adherence, quality of delivery and participant responsiveness (Dane & Schneider, 1998; Dusenbury et al., 2003). In terms of teaching practice conducted by the five participants, an issue of being able to implement the reformed curriculum as designed was revealed. For instance, almost all the teachers had difficulty implementing important aspects of the reformed curriculum such as components of the lesson format, which were not aligned. Similarly, in terms of delivery, teachers spent most of their teaching time explaining concepts rather than engaging students constructively as intended in the curriculum.

These findings (Dane & Schneider, 1998; Dusenbury et al., 2003) share many similarities with the current study, mainly in terms of teachers’ heavy workloads and inadequate understanding of the reform. However, there are several other factors that have affected implementation of mathematics curriculum reform in Bhutan. As argued by Handal (2001) in his studies, a part of the mis-match between intention and implementation may be attributed to teachers and students working on more limited goals than those suggested by curriculum designers. According to Handal (2001), many mathematics teachers are concerned only with students acquiring facts and performing skills prescribed by the syllabus and assessed summatively, rather than broader educational goals. A similar study was conducted by Tarmizi et al. (2010) in Malaysia. They found that although teachers hold beliefs aligned with the curriculum intentions, constraints and demands to fulfil examination needs and coverage of syllabus restrain teachers from its implementation. This is illustrated by comments made by Norbu in one of the post-lesson interviews:

Teaching a new curriculum is simple and really interesting with full of activities and sufficient information in both guidebook and textbook, compared to the previous curriculum, which was very short and complicated in terms of information input, particularly in the textbook. The examples cited are based on students’ life context such as names and places used are all related to the local context. It is made very easy and friendly for the teachers to use it in terms of explanation.
As indicated in the quotation above, despite her concerns in covering the syllabus in time discussed in earlier chapters in conducting the Try This activity, Norbu indicated her positive response towards the nature of the new curriculum compared to the previous curriculum, particularly with the use of context-based examples. However, her knowledge of context-based examples tends to be limited to using local names and places, and she used these in conducting her lessons. Moreover, she was still focussed on teaching through explanation of the concept, rather than on students’ learning activity. This further underlined that her beliefs were Platonist rather than experimentalist, and are therefore mis-aligning with the intention of the new curriculum. The researcher speculates that Bhutanese mathematics teachers are hindered in their ability to effectively deliver the new curriculum for a number of reasons, such as mis-matched beliefs about mathematics, language difficulties, inadequate orientation and an inadequate system in place to conduct follow-up of the reform. The following sub-sections discuss each of these factors in turn.

8.4.1 Mis-match between teachers’ expressed beliefs and practice

Based on the survey, the majority of respondents expressed beliefs which supported a constructivist view of teaching and learning of mathematics, thereby aligning with the intentions of the new curriculum. However, in practice, as discussed in Chapters 5 and 7, the participating teachers were aligned more with Platonist beliefs. They focussed on teachers’ explanation rather than students’ own exploration of ideas. However, the problem observed during the data collection with teachers was a lack of ideas on how to modify the learning activities and make them real to students. Consequently, participating teachers appeared to be quite limited in their knowledge of engaging students productively to work on their own. The teachers’ approach of explaining and conducting the task themselves left students sitting passively doing nothing, as depicted in Figure 8.1.

Figure 8.1. Impact of teacher centred approach.
Teachers seemed to be lacking in terms of engaging students in learning by doing, thereby limiting students in connecting their own understanding of knowledge both vertically and horizontally. For instance, activities such as that reproduced below from Gawa’s lesson were not observed in the majority of lessons.

“I have three A4 sized papers and want to share it equally among four students. What should I do? How? Why?”

Aydin et al. (2009) are among many researchers who have argued that beliefs are considered a major factor affecting teachers’ way of teaching and changing their practice of teaching. Hence, as found in such past studies, mis-match of the teachers’ beliefs and practice is found to be one of the factors hindering the effective implementation of the new curriculum. Beliefs impact on practice but one can only implement beliefs if the conditions are suitable and one has the confidence to do so. For instance, teachers trying their best to cover the syllabus in time for students to prepare for their examinations and score good marks tend to over-rule the implementation of practice consistent with their beliefs, as shared by Phurba in Chapter 7. The same situation was reported by Barkatsas and Malone (2005), who argued that:

Teachers may believe, for instance, that group work is the best environment for exploring mathematical ideas and learning mathematics, but that preparing their students for university-entrance examinations and the pressure to achieve the highest scores possible for each student may keep them from implementing their beliefs into practice (p. 86).

Moreover, as in the situation that prevailed in this study, Barkatsas and Malone indicated the difficulty of introducing and sustaining innovative teaching practices in a system which has solid traditional foundations. They recommended that the expansive social and cultural environment of the classroom may influence teachers’ embraced (and legislated) beliefs about mathematics and mathematics education. Clearly, the culture of schooling in Bhutan appeared to be one constraining factor. One element of this was teachers’ heavy workloads, as discussed in the following section.
8.4.2 **Teachers’ heavy workload**

Despite the current policy of minimising the intake of new teachers, there are still many schools, particularly in remote areas, that suffer teacher shortages. For many years, primary schools in the remote areas were sometimes run by two or three teachers including the school principal. Teachers posted to such schools routinely are exhausted at the end of the day, attending to normal duties including co-curricular activities after teaching hours. To expect teachers to provide quality education is unrealistic when they have to grapple with issues like large classes, heavy teaching loads, numerous activities to carry out daily, as well as normal duties (Ministry of Education, 2007). For instance, in Dragon, it was challenging to get teachers to the post-lesson interviews after school hours, let alone to participate in teaching during school hours. A heavy workload also impacts the capacity for teachers to reflect on practice (Boody, 2008).

Moreover, most of the time, participating teachers were either engaged in co-curricular activities or being called out by the school principal to attend important meetings. In this way, teachers seem to face a challenging time to prepare lessons. For instance, in Takin school, other teachers of primary students’ mathematics were interested in participating in the study but could not due to their heavy workload. The researcher often found it difficult to find times when the teachers were available for interviews. In this way, teachers appeared to be heavily engaged with school activities in and after school hours, as stated by Tempa, during one of the post-lesson conferences:

> Madam, I could not do as I intended due to shortage of time. Moreover, I could not plan my lesson yesterday as my child was sick. So, I prepared this morning only.

From the above statement, one can deduce that teachers are heavily engaged with school and home activities besides teaching, and have little time for lesson preparation or to reflect on their practice. Bantwini (2010) found that the implementation of reform was effected due to teachers’ overload with teaching as well as other responsibilities assigned to them. Besides this, as pointed out by Klein (2001), implementation of a reformed curriculum can be affected if social issues and contextual factors that surround schools are neglected. For instance, contextual factors such as high teacher: student ratios and teachers’ workloads are to be
considered while evaluating the impact of the reformed curriculum (Bantwini, 2010). For instance, in this study, the advocated 1: 40 ratio in almost all the four classes from the two chosen schools (refer to Chapter 5) were still unattainable at primary school level, the level at which learners require more attention and where strong learning foundations are built.

As such, teachers were often found struggling to give individual attention to all students. The findings from Beck et al. (2000), who explored teachers’ beliefs regarding the implementation of constructivism in their classroom, argued that it is essential for teachers to be provided with adequate time and knowledge to modify existing curricular materials to suit the reforms. Moreover, a teacher with a heavy workload may have no time to prepare and seek for further knowledge as demanded in the reformed curriculum, thus leading towards inadequate understanding of the reform, discussed in the next section.

8.4.3 Inadequate understanding of the reform

Cheung and Wong (2012) argued that “teachers’ inadequate understanding of and support for the reform is considered as the top hindering factor to the reform process” (p. 52). In their study on factors affecting implementation of curriculum reform in Hong Kong conducted by Cheung and Wong (2012) found that teachers had a limited understanding of new curriculum. The same findings were clearly seen in this study through three different sources of data, such as post-lesson interviews, analysis of sample learning activities and the video-recorded lessons. There could be numerous reasons affecting teachers’ understanding of the new curriculum. For instance, according to the study conducted by Educational Initiatives and Royal Educational Council (2010), Primary Teacher Certificate graduates are not only deemed to lack content knowledge and but also knowledge of associated learning theories, especially in relation to the new curriculum. Modules addressing mathematics education theories are not included except for two modules on teaching mathematics. As such, Primary Teacher Certificate graduates were not only deprived of content modules but also knowledge of contemporary learning theories related to mathematics education. Hence, according to Bantwini (2010), lack of understanding of reform could act as a hindrance to positive change and implementation.

The predominance of primary teachers with Primary Teacher Certificate qualifications was also shown in the survey demographic information in Table 5.2.
However, in this study, there is not much difference in findings in terms of expressed and manifested beliefs between teachers with a Primary Teacher Certificate and a Bachelor Degree, as indicated in Table 5.20. The teachers with PTC qualification tend to reflect better results than teachers with a Bachelor Degree, particularly in the application of process standards in designing sample learning activities. For instance, in terms of percentage in reasoning, enactive representation and verbal cum written communication, Primary Teacher Certificate holders were slightly higher than Bachelor Degree graduates.

Powell and Anderson (2002) argued that implementation of a reformed curriculum often entails a transformation in teachers’ ideas about the subject matter and the teaching and learning of mathematics. Further, Guskey (2002) claimed that experienced teachers seldom become committed to an innovation or reformed ideas until they have seen them work in their classrooms with their students. For instance, in her lesson reflection, Sangay seemed to have realised the impact of providing insufficient materials for group work. This realisation was highlighted in terms of deeper understanding of concepts. When conducting group work with a sufficient supply of materials, she observed a positive impact on students’ level of understanding, evidenced by her statement:

The majority of students appeared to have understood the concept on equivalent fractions better while involving students themselves in creating equivalent fractions using fraction tiles, as they were able to represent, communicate and connect their knowledge” (Lesson Reflection One).

As argued by Tarmizi et al. (2010), educating students mathematically is more difficult, challenging and complex than teaching them mathematics. It is not only important for teachers’ beliefs to be consistent and to change their beliefs to be in tune with the reforms but equally important for policy makers to support the reformed ideas so that the ideas are enacted in the actual classrooms as intended. For this, timely support must be given to teachers to understand and then implement the intended ideas of reformed materials.

In this study, the majority of mathematics teachers seemed not to have received a proper orientation to the new curriculum. For example, as stated by Phurba during the fifth interview “there was no orientation given on how to use the textbook”. As argued by McLean and Hiddleston (2003), there is a high probability that the
thinking and understanding of Bhutanese teachers related to the implementation of reform are not in tune with the intentions of the new curriculum. Moreover, as described in Chapter 7, when the researcher enquired about the use of the Try This activity as suggested in the new curriculum, some of the participating teachers exhibited a limited idea of conducting such activity. For example, Phurba made the statement, “we were told to conduct such activities only if time permits and we are worried that doing this, we will not able to cover the rest of the plan”. Hence, there is a strong requirement for timely and, if possible, on-going professional development that would ensure that teachers understand what is required of them.

In addition, Norbu and Sangay also expressed their reluctance to be involved with such activity in terms of time, stating “the activity took a lot of time and hampered the timely coverage of the syllabus”. However, in the process, the aversion to conduct such activities reflected teachers’ limited content knowledge and skills. For instance, while Phurba was conducting the Try This activity on fractions as division, he stopped half way through and, when queried, he expressed his concern and the researcher had to help him with the content. Similarly, Roehrig and Kruse (2005), in their findings, indicated that “only through intensive one-on-one professional development, over an extended time period, did some teachers confront their beliefs and embrace the reform-based curriculum” (p. 413).

Although data about the teachers’ content knowledge were not collected directly by the researcher, the interview and observations exposed that content knowledge was also a factor influencing the implementation of the reformed curriculum (Roehrig & Kruse, 2005). Moreover, Hill, Rowan and Ball (2005) have argued that teachers’ deep and flexible understanding of mathematical concepts helped them deliver richer learning prospects for students, such as using better concrete simulations and more meaningful examples. Such positive examples of practices were observed in Norbu’s lesson since she cited and used different examples and approaches, although she tended to be limited in ideas for engaging students constructively. However, in spite of holding reform-based teaching beliefs, teachers in the Bhutanese context are also limited in their knowledge of content, and of planning and conducting a reform-based lesson, similar to findings in studies conducted by Roehrig and Kruse (2005).
8.4.4 **English as a medium of instruction**

As discussed earlier, the new curriculum was inspired by principles outlined by the NCTM’s (2000) standards document which, of course, originated in the United States of America. Furthermore, teachers are expected to deliver these reform ideas in English, creating a challenging situation for teachers teaching mathematics. Adopting a Western approach to curriculum to be implemented by Bhutanese teachers in English with inadequate professional training are drawbacks to effective implementation of the reformed curriculum.

As described earlier, one of the intentions of the curriculum was to encourage teachers to adopt the use of meaningful contexts in the teaching and learning process. To support this intention, a textbook based on the intentions of the new curriculum was written, and a Teacher Guidebook was also prepared in order to assist teachers to implement the curriculum in ways consistent with what was intended. Both the textbook and Teacher Guidebook are written in English. The first component given in the sample lesson plan suggested in the textbook is the presentation of a contextualised learning task for students to explore independently to construct new knowledge, independently, through explorations. However, this type of learning task failed to appeal to students’ interest and thus they found it difficult to understand mathematical language in English. At times, teachers themselves found it challenging to conceptualise the demand of the task. For instance, one of the participating teachers left the task incomplete and moved on to the next activity. Later, when the researcher enquired during the post-lesson interviews, the teacher stated that he did not understand the language. With the help of the researcher, he volunteered to repeat the session and teach the concept again.

Students seem to face a challenging time dealing with the type of learning task such as the Try This activities. Although students learn mathematics in English from the Pre-Primary level of education, the challenge in understanding the learning task in English is considerable. Such a problem is shared in many other non-English speaking countries such as Malaysia, where students are taught mathematics and science in English. In a study conducted in Malaysia, Tan and Lan (2011), found that many students could not answer mathematics questions due to their limited capacity in understanding the questions in English. Similarly, Norbu, during her second post-lesson conference shared her challenge in asking students to solve
mathematics problems in English. She stated, almost desperately, “No, they just cannot do it!” Turning to her colleague, Phurba, she asked, “could you do it? I don’t think you could do it either, right!”

Moreover, Phurba reported in his seventh post lesson interview that “Yes, it is true that the children find difficulty in understanding the word problems”. He found improvement in students when he used other alternatives such as translating the problem into diagrams: then, even the weakest student could understand. The only problem observed with students is that they cannot explain verbally in English. Hence, according to Phurba, language can be a substantial barrier to mathematics learning, especially when that language is not the students’ native language.

Consequently, a teacher is forced to read and explain to students in English before they actually start solving a problem on their own. As stated by Norbu, in her third post-lesson interview, “Somehow, students are habituated with the problem being read by the teacher and this way, they become very dependent…students just keep on murmuring Try This Try This …and not move a pen”. Norbu repeated the same sentiment during her 4th post-lesson interview, stating “Students are able to compare fractions when given in numerical form but not able to explain as they find difficulty in forming in sentences”. Consequently, participating teachers tend to feel the Try This activity is a waste of time, hampering the coverage of the syllabus in time. In the process, such constraints not only affect the implementation of curriculum intentions, but also indicate a lack of understanding of the reformed curriculum.

A similar study was conducted in Malaysia by a team of researchers (Ahmad, Jaaman, Majid, & Rambely, n.d) in regard to the challenge of teaching and learning mathematics in English, that emphasises use of the native language to ensure the knowledge presented can be grasped. They found achievement was much higher when Malay language was the medium of instruction rather than English.

8.4.5 Lack of adequate orientation program
As discussed in Chapter 2, it was intended to conduct an orientation program in several steps, beginning with the ToT (Training of Trainers) program conducted centrally. However, due to budget constraints, the ToT workshops took place in only a few regions and thus many schools were overlooked. As a result, almost all
participating teachers seem not to have received detailed orientation to the new curriculum.

The new Bhutanese mathematics curriculum was presented to the teachers with the expectation that they would know how to implement it with little meaningful orientation. Some of the consequences can be observed in the following quotation from Phurba made during an interview:

…Madam, is it necessary to discuss the answer for the Try This inside the class? Can we switch on to the next, I mean information input? Actually, we were never oriented on how to use it properly…it is only today when madam told us about its purpose…for the past six years, I had been using it in the same way…I mean, just touch a bit and move on to the information input…

There are likely to be many other teachers who were deprived of chances to attend the orientation program and had to teach based on their own principles and knowledge. This type of situation could encourage teachers to fall back to an old transmission approach. Hence, there is every likelihood that existing practices still follow what Subba (2006) has described:

Teachers taught us through dictation, the only pedagogical approach teachers seemed to possess at that time. You had to solve problems by looking at the examples in the textbook, repeated a number of times until you got used to the steps that led to the answer. (p. 26)

A similar kind of strategy was found to have been practised by the five participating teachers in this study. However, compared to the past practice, some improvement was also revealed in terms of using different approaches and teaching aids to help the teachers' to explain the targeted concept. For instance, as discussed earlier, when Norbu tried to explain the concept of fractions, she used various teaching aids such as the children themselves, pictures of fruits, a number line and other diagrams. The only improvement required for her would be to engage students in constructing their own knowledge using those strategies that she intended and used in her class. Moreover, the majority of the primary school teachers have a PTC qualification, as indicated in Table 5.3. They would have missed the opportunity to learn about the learning theories related to the social constructivist approach to the teaching and learning of mathematics, on which the new curriculum is based. Hence, until and unless teachers are made aware of the intended curriculum and are
equipped with key reform ideas, it is unlikely that they will be able to achieve fully the goals of the curriculum.

Currently, one of the main problems in terms of implementing the ideas and intentions of the new mathematics curriculum is the lack of timely professional learning. In the absence of well-planned and ongoing professional development, current teachers adhere to the practices of old that encouraged students to acquire procedural types of knowledge. Moreover, the researcher’s reflection based on her experiences in the system for more than 25 years as a mathematics teacher and educator, and also as a curriculum developer, is that compared to other subjects, professional development programs for mathematics teachers were rarely provided. The lack of professional development has apparently been due to fiscal restraints within the Ministry. As such, it is ironic that having provided very promising curriculum materials, students still have to learn mathematics in the Platonist manner by listening passively to the teacher.

8.4.6 Inadequate system in place to conduct follow up of the reform

A further constraint is the system delay in terms of following up the new ideas. The Bhutanese government has invested millions of dollars in hiring experts and consultants from abroad to share the latest ideas to help update the prevailing education system. However, due to neglect of follow up action, most ideas are diluted. In looking at some of the existing practices in the system, the new curriculum was imposed on teachers without either adequate orientation or systematic evaluation.

As argued by Sztajn (2003), in her studies of adapting reform ideas in different mathematics classrooms, teachers are almost left to face the challenges of reform in mathematics by themselves, thus giving teachers freedom to implement the curriculum in ways feasible to them. In some schools, the new curriculum was in use only for the last few years of primary school. For instance, in Dragon school, the new curriculum was implemented only in 2012. Prior to this, teachers were using another curriculum introduced in the school to be pilot tested by Royal Education Council, to which teachers were more attracted than the new national curriculum. As stated by Tempa during one of the post conferences, “actually, we were asked to use both the new and the other one but we decided to stick to one and we found the other one more friendly and easy as it came in a package with a workbook”. When asked about
the performance of students in terms of quality learning, Tempa found the earlier curriculum better, his reasons being that “problems are easier requiring less time to think and they can just use the method, whereas the new curriculum requires more time to think”. From this statement, one can deduce that Tempa’s teaching practices are shaped more by how much time his lesson takes rather than encouraging students to develop understanding, which takes longer. According to constructivist theory, students learn better if they provided with thought-provoking tasks, requiring more time to think and construct their own knowledge.

Nevertheless, having expressed their opinion about the advantages of the old curriculum, teachers took their own initiative to adopt the new curriculum mainly due to the national testing system where questions were set according to the new curriculum. The reputation of the school and the performance of the teachers were measured by students’ test results. This is a further indication of the disconnect between beliefs and practices: teachers knew their school’s reputation and their own careers depended on the students’ results. Thus, such cultural issues in the school could contribute to the creation of a gap between expressed and manifested beliefs. Similarly, physical structure and the classroom environment could possibly add another issue, as presented in the next section.

8.4.7 Physical structure and the classroom environment

The results of the current study indicate that most Bhutanese classrooms are likely be dominated by a traditional physical arrangement. The physical structure of the classroom can add to the quality of curriculum implementation. As argued by Wiske and Levinson (1993), the details of school life also affect teachers’ abilities to incorporate new curriculum and practices. For instance, the classroom structure and environment in Takin and Dragon appeared to be traditional and thus suited to a traditional approach to teaching mathematics with students sitting in rows. One classroom was an exception in that it had students arranged in groups. Nevertheless, in all observed classes, most of the activities carried out were to be performed individually, eliminating the opportunity for sharing of ideas through interaction and verbal communication. Apparently, none of the educational leaders and management team seems to have realised the unfavourable layout for learning mathematics. Hence, the existence of such a physical structure in the classroom limits opportunity for both teacher and students to practise as intended in the new curriculum.
8.4.8 Practices of formative and summative assessment

Contextual issues influence teachers’ assessment practices. The existing Bhutanese education system does value students’ understanding, but student performance is mainly determined by their marks scored in examinations. In recent times, the Bhutanese education system has begun to acknowledge the importance of formative assessment and has taken initiatives to introduce such strategies in schools. Notwithstanding these recent moves as expressed in the reformed curriculum, teachers find little time to spend in organising appropriate materials to conduct formative assessment during the teaching session.

Evidence from this study suggests that summative assessment continues to dominate the assessment strategies adopted in at least the classes studied. Thus, despite their thoughtful ideas and strategies, participating teachers were predominantly concerned with completing their plan of a lesson so that they could cover the syllabus in time, as shared by Norbu and Phurba during one the post-lesson interviews. They had no other choice but to neglect monitoring students’ level of understanding, thus hampering the quality of learning. Ultimately, teachers are forced to rely on a lecturing style to achieve student learning. Hence, teachers tend to avoid monitoring learning or tailoring instruction to meet students’ learning needs to cover the syllabus in time for students to prepare for their examinations.

8.5 CHAPTER SUMMARY

The overall findings from the survey (macro-phase level) in terms of beliefs were encouraging, with a high percentage of respondents supporting experimental beliefs aligned with the intentions of the new curriculum. However, there was a contradictory result in the lesson planning and observations conducted in both the macro and micro-level phases of the study. Teachers’ practices contradicted their expressed beliefs regarding curriculum intentions and depicted Platonist beliefs, where teachers spent more time explaining the concept in detail than having students explore knowledge and ideas on their own. These findings were indicated in Table 8.2, and reflected consistently in all the data collected. Therefore, as stated by Berman and McLaughlin (1976), “the bridge between a promising idea and the impact on students is implementation, but innovations are seldom implemented as intended” (p. 349). Several constraints emerged from the data which limited teachers
implementing the new mathematics curriculum. Some of the most likely ones are illustrated in Figure 8.2.

*Figure 8.2. Constraints influencing the implementation of reformed curriculum.*

The theoretical contribution from this study tends to confirm that it is not always true that teachers’ beliefs shape classroom practice, prior to attending to the factors indicated in Figure 8.2. The findings from this study appeared to have revealed a gap between teachers’ expressed and manifested beliefs. The teachers’ practices are manifested in ways that draw on entrenched and not expressed beliefs, thereby showing a mis-alignment between the intentions of the curriculum and its implementation.

The teachers appeared to be limited in practising what they knew and believed. For instance, the majority of the respondents said they believed that students are capable of constructing their own knowledge but in practice, they are not able to engage and encourage students to do so. Handal and Herrington (2003) argued that a mismatch between curriculum goals and teachers’ belief systems is a factor that affects current curriculum change in mathematics education. In this way, the beauty of learning mathematics tends to be missing in Bhutanese mathematics classrooms. As stated by McLean and Hiddleston (2003):

…The basis of true learning is understanding and not rote repetition of formulae…thus the student should be at the center of the learning process, not as a receptacle to be filled by the teachers, but as a human being valued for his/her individuality and for the unique contribution he or she can make to the wealth (spiritual as much as economic) of the nation. (p. 7)…
A seed of Gross National Happiness (GNH) can be germinated in the mathematics classroom, when students achieve deep understanding and, in the process, enjoy learning mathematics. Mathematics is another form of language and learning that is critical to Bhutan to develop a love or understanding of mathematics and its application in everyday life, to enhance the sum total of GNH. Aligning with this idea and to encourage teachers to implement the intended curriculum efficiently, the authorities could develop a policy which aligns with elements indicated in the model presented in Figure 8.2.
Chapter 9: Conclusions

This chapter discusses the significance and contributions of this study to theory and practice in the field of mathematics curriculum reform. As reported by VanBalkom and Sherman (2010), Bhutan has embarked on a comprehensive education reform process, with teachers and teacher education at the centre of a number of initiatives. This change is mainly to address the great need for human resources in a developing country, particularly when knowledge of mathematics is fundamental. As a part of this policy, the Ministry of Education has taken the initiative to reform the Bhutanese school mathematics curriculum. The agenda is occurring in a context where teachers have had long-term exposure to a mathematics curriculum that was never specifically designed for Bhutan. Moreover, beliefs about teaching and learning have been influenced by centuries of cultural beliefs and practices grounded in Buddhist philosophy.

However, the implementation has not been accompanied by any comprehensive evaluation of the uptake of the reformed curriculum in Bhutanese classrooms. Therefore, on a small scale, this study has presented the current situation on how the standards-based new mathematics curriculum is being implemented in Bhutanese classrooms. The data were collected and analysed to answer the following Research Questions:

Question 1: What are the beliefs of Bhutanese primary teachers about mathematics teaching?

Question 2: What are Bhutanese primary teachers’ planning and classroom practices in teaching mathematics?

Question 3: To what extent are mathematics teaching practices aligned with the curriculum intentions?

Question 4: What influences the implementation of the curriculum intentions?

To answer these Research Questions, the study was conducted in two phases, which
in this thesis, have been described as the macro and micro-level phases. The findings of the two phases of the study were reported in Chapters 5, 6, 7 and discussed in Chapter 8. The vast majority of survey respondents expressed beliefs about mathematics and its education which aligned with social constructivist principles about teaching and learning of mathematics. These views reflected the philosophy of the new mathematics curriculum. However, in practice, the sample of learning activities was mis-aligned with the intentions of the curriculum, which suggested that there was also a discrepancy between the participants’ expressed and manifested beliefs. Such a misalignment was also evident in the micro-level analysis of the teachers’ classroom practices. Based on the data, suggestions regarding factors contributing to this misalignment were made. This chapter now addresses the significance of these findings and how they contribute to understanding of mathematics education, and reveal the constraints that impinge on the process of implementing a reformed curriculum.

9.1 THEORETICAL CONTRIBUTION

This section focusses on the main theoretical contributions of this study in terms of both the host country and more broadly the world of mathematics education. Since this study is the first of the kind conducted in the history of mathematics education in Bhutan, the result is expected to make a useful contribution to educating students mathematically and implanting a seed of Gross National Happiness in the mathematics classroom. As illustrated in Figure 9.1, GNH, being the national philosophy, is the main foundation of development activities of the country, including the national policy on general education. The national policy on mathematics education adopts the best possible learning theories to be included in the reformed mathematics curriculum. The expectation is that by adopting a reform-oriented curriculum, mathematics teachers can implement the teaching of mathematics in a way that creates enjoyment, satisfaction and engagement among students. Hence, the ultimate goal of mathematics teaching is to help contribute to bringing enjoyment to learning mathematics. Students who enjoy mathematics are more likely to progress to higher levels of mathematics and are expected to be happy and successful citizens, thus equipping the country to engage in sustainable and equitable socio-economic development, especially in those technological and creative industries that are grounded in mathematics. This outcome addresses one of
the pillars which define GNH. Early intervention is important and thus this study has addressed the reform process in primary schooling, which, as Fillingim (2010) has pointed out, is a missing focus in research studies across the world.

Figure 9.1. GNH related goals in mathematics education.

Although the vision is clear, there are various factors to be considered in the process of achieving the goal of implanting GNH in mathematics classrooms. One of the most important factors to be considered is the teachers. As revealed from this study, having a dynamic curriculum alone is not enough to ensure successful implementation without reform-oriented teachers with a sufficient supply of teaching and learning materials. The findings of this study confirm much research about the difficulty of implementing change (Fullan, 2007). Many of the constraints identified by Fullan in Western contexts and by others in developing contexts (Bulut, 2005) are confirmed in this study. Policy initiatives such as reforming curricula can set unrealistic expectations, which involve changes in beliefs and knowledge (Schweisfurth, 2011). Challenging and changing teacher beliefs has been established as essential in the reform process, but the difficulties and strategies are well documented (Timperley & Robinson, 2001). Inconsistency and lack of clarity in the communication of reform is one constraint that is evident in the Bhutanese context. Limited support for teachers and lack of time in preparing lessons are also well documented factors in the implementation of the Bhutanese curriculum.
For success, senior policy makers must make clear how reformed teaching can contribute to GNH, and make sure all the relevant elements are present to enable students to gain maximum benefit from learning mathematics. A reform oriented teacher, helping students build a strong foundation in learning mathematics meaningfully, could be considered as planting a seed of GNH in the mathematics classroom. A student learning mathematics in such a GNH atmosphere would be expected to perform well not only in primary level but also to carry on the same spirit at higher levels of education and could be in a position to help build the nation towards GNH. At the same time, high achieving mathematics students can be in a position to help fill the gap in human resources, as discussed in Chapter 2. In this way, the findings from this study may contribute towards the building of a happy and successful nation but also may in a small way achieve a bigger goal in terms of educating students from any nation in this competitive world.

The main focus of this study was primary level education, to help build a strong foundation of learning mathematics to be used constructively later in the field of practice. This point is strongly supported by Fillingim (2010), who argues that currently in the world of mathematics education, most studies of this type have been conducted in higher-level classes. Hence, in the process of investigating the alignment of curriculum intentions with its implementation, the researcher has developed the 3+1 framework, introduced in Section 3.3, to help analyse the practice of any curriculum reforms in the mathematics classroom. With the help of this framework, anyone interested to conduct research on the alignment of intentions and implementation of the reformed curriculum could be in the position to use it.

9.2 PRACTICAL IMPLICATIONS

With the introduction of the New Approach to Primary Education (NAPE), the new mathematics curriculum was not the first attempt to change practices in primary level education. NAPE was another intervention originating from Western ideas based on the social constructivist view of teaching and learning. This approach was introduced in the early 1980s in the Bhutanese primary schools in teaching general subjects, including mathematics. However, due to several factors such as social and cultural issues of the schools, this program could not be materialised as intended. The large number of students and the lack of adequate materials were some of the main factors that caused the failure.
9.2.1 **Challenges**

Despite the promising features of NAPE, it has not been very successful so far as implementation is concerned. Although the appropriate beliefs were there, the skill to implement them is not clearly evident. However, awareness of the student-centred approaches was created among policy makers by NAPE, with implications for future changes in teacher education.

There has been some concern among educators that developing countries such as Bhutan have attempted to adopt Western curricula in ways that are unsuited to their context. For one, the preparedness of students for formal education varies considerably between developed countries and developing countries. It could be the case, for example, that many children in Western countries are quite numerate before they start school and are proficient in basic mathematical ideas. Curricula that are age-based may be inappropriate although as in Australia, a child in Class 4 in Bhutan may have had five years of formal schooling including the preparatory year but without having had the opportunity to develop mathematical concepts in the English language.

Although it is well established that beliefs influence practice, this study has shown that identifying which beliefs teachers hold is challenging. At one level, Bhutanese teachers express beliefs that are consistent with understanding constructivist, student-centred active learning. They also exhibit views of mathematics that are conducive to constructivist-informed approaches to teaching. The majority of the survey respondents supported experimentalist beliefs that aligned with the intentions of the new curriculum. However, in an analysis of the actual practice of the teachers, it was evident that teachers practised traditional transmissive chalk and talk methods. They ignored instructional guidelines which encouraged them to engage students in dialogue and discourse about their understandings. Although there was some use of appropriate and context-based examples to help explain mathematical concepts, rich complex problems were largely absent from their teaching strategies. Nevertheless, there were signs that their practices were unlike those of traditional Bhutanese teachers, who have often failed to connect mathematical concepts to what children experience outside the classroom, something which is now considered essential if the aim is to make the learning of mathematics meaningful.
Almost all participating teachers were found to be good in explaining the taught concept using limited context-based examples. In this case, seeds of constructivist ideas were sown, with teachers being encouraged to choose appropriate examples related to their students’ real-life activities. However, the study revealed little evidence that teachers engage students to explore problems on their own, or to work collaboratively as they attempt to solve problems and give them enough time to think creatively about the problems. Moreover, formative assessments were rarely conducted. In terms of practice, all participating teachers demonstrated Platonist beliefs about mathematics. These findings highlighted the challenges faced in Bhutan with the introduction of curricula that are aligned with contemporary theories of teaching and learning, particularly in terms of delivery (engaging students constructively). Hence, findings from this study tend to support the conclusion that reform takes time to be materialised as intended.

This experience indicates that the prevailing situation was not favourable for teachers to implement new ideas in terms of time and space to reflect on their beliefs about the teaching and learning of mathematics. For success in implementation of the reform curriculum, teachers’ beliefs about the current reforms need to be reflected upon, supported and challenged. Support can be rendered in the form of providing sufficient time for teachers to reflect on their beliefs about mathematics and provide adequate space for them to practise accordingly. Otherwise, there is a tendency for teachers to continue traditional practices in the privacy of their classrooms, and the implementation process as a result would be a waste of energy and resources (Handal & Herrington, 2003).

Herbel-Eisenmann et al. (2006) argued that although a teacher’s comprehensive orientation toward teaching profoundly influences instructional practices, contextual factors can prompt local changes in teachers’ enacted instruction. It might be possible that the documentation of those teachers’ experiences, in this present study could precipitate changes being made to pre-service teacher-education curricula, so that these would place greater emphasis on the provision of appropriate rich learning activities for in-service mathematics teachers.
9.2.2 Recommendations

As argued by Stigler and Hiebert (1999), school learning will not improve significantly unless teachers are given the opportunity to advance their expertise by increasing the effectiveness of the methods they use. By aligning ideas from Stigler and Hiebert’s (1999) research with the findings from this study, provides the following recommendations.

**Recommendation 1: Long term ongoing professional learning activities**

The results of the current study indicate that the teachers lacked adequate professional learning on the implementation of the new curriculum. It is important to note that persons teaching mathematics in Bhutanese primary schools do not have many opportunities to gain access to professional learning activities. This limitation was clearly indicated in terms of engaging students constructing their own ideas and knowledge. Several studies have revealed that effective curriculum reform involves a transformation at the individual teacher level focussing on teachers’ beliefs through appropriate and timely ongoing professional learning activities (Harris, 2003; Hopkins, 2007). Powell and Anderson (2002) argued that to have a successful implementation of reformed curriculum materials, teachers are required to attend timely and adequate professional learning activities, and transform ideas in terms of their understanding in related subject matter such as the pedagogy of mathematics. Similar findings were presented by Keys (2007). Hence, policy makers within the Ministry of Education should be in a position to devote considerable resources in ongoing professional learning activities in providing support, conducting monitoring and promoting teacher alliance within schools and other learning organisations.

Stigler and Hiebert (1999) identified the mismatch which often occurred between intended and implemented curricula, and argued that this was the result of inadequate knowledge of the range of possible teaching methods rather than of weak teachers. They referred to the kinds of teaching used to implement the intentions of curriculum more effectively to help students develop a deeper understanding of mathematics. In order to improve students’ academic achievement significantly, teachers are expected to have appropriate knowledge and skills in teaching mathematics that match the intentions of the curriculum (Education Sector Review Commission, 2008). Thus, there is much to be done at the school level. Long-term ongoing professional learning activities are recommended for existing teachers to
help them change their beliefs to practices aligning with the intention of the new curriculum, as has occurred in countries such as Japan and Indonesia.

Considerations in relation to the findings of the study described in this study could contribute to further improvement of educational policy and assists other mathematics teachers facing the same problems enacting the intention of a new mathematics curriculum. It may convince policy makers of the need for more funding, or influence strategies around the way new policy is implemented. Findings from this study reveal that there is a strong requirement for teachers to move from the traditional model of professional development of just providing knowledge and skills, to the inclusion of reflective practice that makes a connection with existing beliefs. Therefore, it is recommended that the Bhutanese government and teacher-training institutions provide pre-service teachers with a clear understanding of how they want teachers to understand the concept of a standards-based curriculum and how this should be enacted at the classroom level.

**Recommendation 2: Curriculum materials written in appropriate language**

Well-designed curriculum materials such as teachers’ guidebooks and students’ workbooks, based on the intention of the new curriculum and relevant to the Bhutanese context, may not have a productive effect if delivered in a foreign language. The finding of the current research demonstrate that English as the medium of instruction generates constraints in terms of understanding concepts, particularly from written texts. One way of solving this problem using existing materials could be to reduce text to make it simpler, and to place greater emphasis on visual forms. An alternative suggestion is to train the teachers to implement the given word problems in more simple and realistic forms, such as making them more dramatic and enriching for students. Having students as reviewers of these materials may help to make them more authentic and therefore more appropriate to their cognitive readiness level.

**Recommendation 3: Strengthen the focus of assessment for learning**

The findings of the current study show that there were few chances for students to be actively involved in the learning process. The focus for teaching and learning at present was on covering the syllabus in time. In the process, any focus on essential knowledge tended to disappear, and the focus became more one of drilling procedural knowledge and helping students pass the examinations. There were
relatively few activities which engaged students in meaningful learning and in providing teachers with opportunities to plan and implement effective formative assessments. According to the new curriculum, assessment for learning is expected to occur in parallel with teaching, so that the teacher can facilitate students’ learning. Therefore, all concerned authorities and teachers are expected to place more importance on reducing assessment of learning through pen-and-paper methods, and increasing assessment for learning to align with the intentions of the reformed curriculum and the social cultural context of education in Bhutan. It is important for them to realise that to introduce assessment for learning without consideration of the social cultural context in which it is implemented, is a major challenge.

**Recommendation 4: Reformed curriculum in pre-service teacher education**

As indicated by VanBalkom and Sherman (2010), most of the faculty members in the two colleges in Bhutan use lectures as their main strategy in teaching and do not model different teaching strategies, particularly those informed by social constructivist perspectives. On the basis of the evidence in this study, failing to do so leads to the production of poorly prepared graduates and this in turn contributes another generation of mediocre teachers to repeat the cycle (VanBalkom & Sherman, 2010). It is important for all concerned officials to realise that mathematics education is a field which can develop properly only after a rigorous teacher-education program based on both theory and aligned teaching methodology (Wittmann, 2005).

Therefore, the Bhutanese government, instead of investing millions of dollars in hiring one-stop experts and consultants from abroad, should fund long-term professional learning activities through pre and in-service teacher training programs.

### 9.3 SUGGESTIONS FOR FUTURE RESEARCH

Ample opportunity is seen for future research related to the existing practices of the teaching and learning of mathematics in Bhutanese schools. Some of the questions that emerged in this study suggest further research on the following:

- Explore the relationship between intention and implementation of the new curriculum further with those teachers who have attended orientation programs on the new curriculum. The purpose of this research would be to study the quality of the implementation of the new curriculum and compare it with the results of the current study. In this present study, only
one of the participating teachers attended a week’s orientation program on
the new curriculum. However, in terms of practice, despite this training,
her practices were not aligned with the curriculum intentions.

- Undertake a study focussing on the reactions, reception and performance
  of students associated with the intentions of the new curriculum. This
  present study focussed only on the teachers. An important issue is whether
  the students in the teachers’ classes were ready for an imported curriculum
  theory.

- Conduct mathematics lesson using the local language and compare the
  effects on student learning of these lessons with the effects of lessons
  conducted in English.

9.4 LIMITATIONS

The researcher recognises that this study has limitations. Although the philosophical
aim of this study was to help students learn mathematics more meaningfully, there is
no direct involvement of students as research data sources. Other than the collection
of a few artefacts from students in the process of lesson observations, there were no
other direct activities where students were required to participate in this study. The
main focus of the study was on teachers with the intention of helping teachers to
facilitate students’ learning in effective ways. One of the main limitations was the
exclusion of students in exploring the impact of the new curriculum. Hence, a study
focussing on the reactions, receptions and performance of the students on the change
in teaching mathematics should be explored.

The research was also limited in scope that it explored the implementation of a
new curriculum in a small number of mathematics classrooms in primary schools in
Bhutan. Its findings should not be generalised to other contexts. Although the
schools in which research data were collected may be representative of primary
schools in Bhutan, it still cannot be claimed that the findings are consistent across all
primary schools in Bhutan. However, individual teachers, policy makers and
researchers are able to consult this research as evidence to inform strategic or policy
directions. The study provides findings that may help in explaining other similar
situations.
The duration of the data collection of this research was also a limitation of the study. In the initial plan, the data were to be collected in two different phases, the macro preceding the micro-phase level study. The collection of data for the micro-phase was to take place after determining the results from the macro phase. However, in reality, due to a number of unavoidable factors which were beyond the control of the researcher, a fine grained analysis of the survey data was not feasible prior to the start of the second phase. Moreover, data collection occurred during the time when the whole nation was busy preparing for the election of national councils and a new government. Unfortunately, many of the school teachers were involved in the election, including the research participants. Hence, the data collection for these two phases of study happened almost simultaneously. Moreover, due to the time constraint for the researcher, who was enrolled as a full time external student and had to bear the pressure of both the studies and duties simultaneously.

Another possible limitation is the influence of the researcher on the participants. For instance, initially, the researcher intended to include professional learning activities in the process of data collection with a team of primary mathematics teachers. However, this plan was not carried out due to the shortage of free periods for the teachers. Finally, although, the researcher made very clear the aims and objectives of the study and ethics related issues in the beginning of the data collection, some limitations occurred in terms of the student-teacher relationship between the researcher and the participating teachers. The researcher had been working as a mathematics lecturer in one of the Teacher Training Colleges for more than a decade, so there was high probability that the majority of the participants could have been her students. Moreover, during the time of her data collection, the researcher had been holding the post of Dean in the College, which may have resulted in participants being prepared to share what they really believed.

9.5 FINAL REFLECTION: LESSONS LEARNT

I, the researcher as a former student, a teacher, and now as a mathematics educator in Bhutan have long been concerned at the challenge of getting Bhutanese school children to be well educated mathematically. The aim was to find a way of sowing a seed by which GNH would blossom in mathematics classrooms, so that students would enjoy and find meaning in the learning of mathematics.
As a mathematics teacher and an educator, it was always my concern to become more informed about how to help students to learn mathematics with enjoyment, confidence and understanding, so that they would develop the power to link what they learned to their everyday activities. When I was a school girl, I dreaded mathematics lessons. Everything seemed to revolve around learning a package of rules and formulae, and there was always the threat of punishment if rules were not thoroughly memorised. The main purpose of mathematics lessons seemed to become prepared so that you would pass the final examinations, and thereby be promoted to the next level.

However, by destiny, I became one of the mathematics teachers conducting mathematics lessons, who could not help but follow the steps of my seniors in teaching mathematics through chalk-and-talk methods. This emphasises the powerlessness of individual teachers to change their practices when the culture of the school and the majority of teachers are comfortable with traditional approaches. Thus, the experience I gained from my training did not help me, as I was under lecturers with a similar background. I could not remember experiencing any type of activity associated with the constructivist approach in learning mathematics, as discussed in Chapter 3. Nevertheless, as was related in Chapter 2, a year-long course in Mathematics Education from the University of Leeds changed my beliefs about mathematics and mathematics education. The credit goes to my tutor, who was a real role model of mathematics. Since then, I have always tried to share the ideas gained from that time directly with my students and fellow colleagues in the mathematics departments, and indirectly through a mathematics manual developed for every primary class.

Thus, having contributed ideas at the policy level, my concern has been to help students learn mathematics more meaningfully did not stop there. In fact, I became increasingly concerned about the quality of Bhutanese teachers’ knowledge of mathematical content, their beliefs about how mathematics should be taught, and about any changes to their actual mathematics teaching practices which occurred as a result of their participation in professional learning activities. In fact, it was found that although most of the participating teachers held beliefs that were consistent with the new curriculum, their teaching practices were not consistent with those beliefs. This finding was explored further in the micro-level phase, where it was observed
that almost all participating teachers had problems delivering the intentions of the new curriculum. The main message from the study was that today’s students experience mathematics lessons which were not much different from the lessons which I experienced in my own school days.

I have also found that having a dynamic and promising curriculum is insufficient if teachers involved in delivering the curriculum are not educated accordingly. Along with reformation of the teachers, there should be reformation in the context of the learning atmosphere and infrastructure in tune with the intentions of the new curriculum. In addition, special consideration should be made in terms of the language used in delivering the reformed curriculum, and the meaningful learning of mathematics to students should be promoted. Eliminating constraints is expected to allow a seed of GNH to be implanted in the mathematics classroom and in turn produce more mindful and productive citizens. Hence, the vision of the new mathematics curriculum could be implemented effectively in educating students mathematically, through collaboration of the elements indicated in a model of Figure 9.2.

![Figure 9.2. A vision for a new mathematics curriculum.](image)

Such a vision, and its effective implementation, would produce a citizenry able to contribute to Bhutan’s Gross National Happiness through mathematical understanding.
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Appendices

APPENDIX A: ETHICS DOCUMENTS
PARTICIPANT INFORMATION FOR QUT RESEARCH PROJECT

Investigating the gap between curriculum intent and practice in Bhutanese Primary School Mathematics Classrooms
QUT Ethics Approval Number 1300000027

RESEARCH TEAM

Principal Researcher: Phuntsho Dolma – PhD student – QUT
Supervisors: Dr David Nutchey – Principal Supervisor – QUT
Dr Gillian Kidman – Associate Supervisor – QUT

Description

The purpose of this research is to investigate the gap between the intention and the implementation of the new mathematics curriculum in Bhutanese mathematics classrooms. This research is not a critical examination of teachers’ practice in teaching mathematics in Year 5, but instead a detailed exploration aimed at understanding the alignment between the intention and the implementation of the new Bhutanese mathematics curriculum in the classroom.

You are invited to participate in this project because you are a mathematics teacher teaching in one of the selected schools that have agreed to participate in the study.

Participation

Your participation will involve responding to the attached questionnaire. As well as gathering some demographic information, the questionnaire covers your opinions of new the Bhutanese primary mathematics curriculum and its implementation and also your beliefs towards the teaching and learning of mathematics. Included in this package is a readymade envelope with which you can return the completed questionnaire to the researcher.

Your participation in this research is entirely voluntary. Your decision to participate or not participate will in no way impact upon your current or future relationship with your school or with Ministry of Education, Bhutan.

Expected benefits

This research is intended to provide information (including the development of theory that could be applied beyond the schools participating in the study) that may be of assistance in improving the alignment of the intention and implementation of the new mathematics curriculum. It may assist:
• The Ministry of Education, Bhutan, by providing information about what is, and could be, occurring in schools that may inform future policy development.

• The general public, by the provision of more information about the intention of the new mathematics curriculum, especially in Bhutanese primary schools.

• The administration of the participating schools, by providing opportunities to reflect on the effectiveness of their strategies in handling the content of the new mathematics curriculum more appropriately and meaningfully.

• The teachers involved in the study, by providing opportunities during the conduct of the study to reflect on the effectiveness of their strategies in helping the students understand the taught mathematical concepts more deeply and meaningfully.

Risks

The most significant risk arising from your involvement in this study is inconvenience. These include disturbance of your time in answering the questionnaires. It is possible that you may be discomforted by the process of reflecting on your practice as a teacher or on the policies and practices of your school.

Privacy and Confidentiality

Your response to this survey is entirely anonymous, and so in no way will you nor your school be identifiable in the findings of this research project.

Consent to Participate

The return of the completed questionnaire is accepted as an indication of your consent to participate in this project.

Questions / further information about the project

If you have any questions or require any further information please contact one of the research team members.

Phuntsho Dolma – PhD student
School of Mathematics, Science, Technology Education – Faculty of Education – QUT
+61 7 3138 5557
phuntsho.dolma@student.qut.edu.au

Dr David Nutchey – Principal Supervisor
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+975 1760 3759
d.nutchey@qut.edu.au

Concerns / complaints regarding the conduct of the project

QUT is committed to research integrity and the ethical conduct of research projects. However, if you do have any concerns or complaints about the ethical conduct of the project you may contact the QUT Research Ethics Unit on +61 7 3138 5123 or email ethicscontact@qut.edu.au. The QUT Research Ethics Unit is not connected with the research project and can facilitate a resolution to your concern in an impartial manner.

Thank you for helping with this research project. Please keep this sheet for your information.
Subject: Introducing a survey questionnaire for research

Dear Sir/Madam

Please let me introduce myself. I am Phuntsho Dolma, Dean of Student Affairs in Paro College of Education under the Royal University of Bhutan. I am currently a PhD candidate at the Queensland University of Technology (QUT), Brisbane, Australia, under RCSC-QUT scholarship. The research topic that I have chosen for my thesis is 'Investigating the gap between curriculum intent and practice in Bhutanese Primary School Mathematics Classrooms'. The research explores mathematics teachers’ practices and how they align with the intent of the newly introduced mathematics curriculum in the country. Permission to conduct this research activity has already been obtained from the Ministry of Education and also from the Principal of your school.

The first phase of this study involves the participation of mathematics teachers from randomly selected Primary Schools. In the first phase, which I am inviting you to participate in, teachers will respond to a short anonymous survey questionnaire. The content of the questionnaire relates to teachers’ beliefs and practices regarding the nature of mathematics and its teaching and learning in relation to the newly introduced mathematics curriculum. The study is not a critical examination of a participating teacher’s practice and pedagogy but is instead a detailed exploration of the alignment between the intentions and the implementation of the new mathematics curriculum in the country.

Any data collected as part of this study will be stored securely in accordance with QUT’s Management of Research Data Policy. This study has received approval from the QUT Human Research Ethics committee (Approval number 1300000027) and thus meets the Australian National Health and Medical Research Council guidelines for research on humans.

Therefore, I would be very grateful if you could kindly spare a few minutes of your precious time to answer the enclosed questionnaire, please. In addition, a more detailed information statement sheet is provided to give further information regarding this study. The completed questionnaire can be returned by regular post using the included envelope, please.

Your kind support and consideration is very much appreciated. Please contact me if you have any questions or issues needing clarification.

Thanking you

Yours faithfully

(Phuntsho Dolma)

Researcher (PhD Candidate)
School of Mathematics, Science & Technology Education
Faculty of Education
Victoria Park Road, Kelvin Grove
Queensland University of Technology, Australia
Telephone: 0434 405 973 / 975 1760 3759
Email: phuntsho.dolma@student.qut.edu.au
APPENDIX B: SURVEY QUESTIONNAIRES

Investigating the gap between curriculum intent and practice in Bhutanese primary school mathematics classrooms

Phuntsho Dolma, PhD Student

Survey Questionnaire

This survey questionnaire is in three sections.
Please complete each section to the best of your ability.
The return of the completed questionnaire is accepted as an indication of your consent to participate in this project.
Section I: Demographic information

Please place a tick in the appropriate box.

1. Gender
   - □ Male
   - □ Female

2. Age level
   - □ 21 – 30 years
   - □ 31 – 40 years
   - □ 41 – 50 years
   - □ 51 – 60 years

3. Qualification
   - □ Primary Teacher Certificate
   - □ B.Ed Primary (Mathematics)
   - □ B.Ed Secondary (Mathematics)
   - □ Any other
   (.................................................)

4. Years of teaching experience
   - □ 1 – 3 years
   - □ 4 – 7 years
   - □ 8 – 19 years
   - □ 19+ years

5. Location of your school
   - □ Urban
   - □ Semi-urban
   - □ Rural
   - □ Remote

6. Classes currently taught
   - □ Class PP-III
   - □ Class IV
   - □ Class V
   - □ Class VI

7. Position in school
   - □ Teacher
   - □ Vice Principal
   - □ Principal
   - □ HOD

8. Class size
   - □ 10 – 20 students
   - □ 21 – 30 students
   - □ 31 – 40 students
   - □ 41 students and beyond
Section II: Scaled responses

Please note that the following 27 questions (in four parts) just seeks your opinion, that is, there is not a correct answer.

For each question, please place a tick in the box for the response that you think is most appropriate (i.e., strong disagree, disagree, not sure, agree, strongly agree).

**Part A: Mathematics**

<table>
<thead>
<tr>
<th></th>
<th>Strongly disagree</th>
<th>Disagree</th>
<th>Not sure</th>
<th>Agree</th>
<th>Strongly agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Mathematics is computation.</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2. Mathematics is problems given to children should be quickly solvable in a few steps.</td>
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<tr>
<td>3. Mathematics is the dynamic searching for order and pattern in the learner’s environment.</td>
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<tr>
<td>4. Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking.</td>
<td></td>
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<tr>
<td>5. Right answers are much more important in mathematics than the ways in which you get them.</td>
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<tr>
<td>6. Mathematics is a body of knowledge isolated from the rest of knowledge.</td>
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</tr>
</tbody>
</table>
### Part B: Mathematics Learning

<table>
<thead>
<tr>
<th></th>
<th>Strongly disagree</th>
<th>Disagree</th>
<th>Not sure</th>
<th>Agree</th>
<th>Strongly agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td>Mathematics knowledge is the result of the learner interpreting and organizing the information gained from experiences.</td>
<td></td>
<td></td>
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<tr>
<td>8.</td>
<td>Children are rational decision makers capable of determining for themselves what is right and wrong.</td>
<td></td>
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<tr>
<td>9.</td>
<td>Mathematics learning is being able to get the right answers quickly.</td>
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<tr>
<td>10.</td>
<td>Periods of uncertainty, conflict, confusion, and surprise are significant part of the mathematics learning process.</td>
<td></td>
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<tr>
<td>11.</td>
<td>Young children are capable of much higher levels of mathematical thought than has been suggested traditionally.</td>
<td></td>
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<tr>
<td>12.</td>
<td>Mathematics learning is enhanced by activities which build upon and respect student’s experiences.</td>
<td></td>
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<tr>
<td>13.</td>
<td>Children construct their own mathematical knowledge.</td>
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</tr>
</tbody>
</table>
### Part C: Mathematics Teaching

<table>
<thead>
<tr>
<th></th>
<th>Strongly disagree</th>
<th>Disagree</th>
<th>Not sure</th>
<th>Agree</th>
<th>Strongly agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>14. Teachers should provide instructional activities which result in problematic situations for learners.</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>15. Teachers or the textbook – not the student – are the authorities for what is right or wrong.</td>
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<tr>
<td>16. The role of mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge.</td>
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<tr>
<td>17. Teachers should recognize that what seems like errors and confusions from an adult point of view are children’s expressions of their current understanding.</td>
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<tr>
<td>18. Children’s development of mathematical ideas should provide the basis for sequencing topics for instructions.</td>
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<tr>
<td>19. It is unnecessary, even damaging, for teachers to tell students if their answers are correct or incorrect.</td>
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<tr>
<td>20. Mathematics skills should be taught in relation to understanding and problem solving.</td>
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</tr>
<tr>
<td>21. Mathematics instruction should be organized to facilitate children’s construction of knowledge.</td>
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</tr>
</tbody>
</table>
### Part D: New Mathematics Curriculum

<table>
<thead>
<tr>
<th></th>
<th>Strongly disagree</th>
<th>Disagree</th>
<th>Not sure</th>
<th>Agree</th>
<th>Strongly agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>22. The new mathematics curriculum is realistic and relevant.</td>
<td></td>
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<tr>
<td>23. The new mathematics curriculum is challenging but interesting for students.</td>
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<tr>
<td>24. The new mathematics curriculum is well connected within and beyond the curriculum.</td>
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<tr>
<td>25. The new mathematics curriculum is complicated and difficult to follow.</td>
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</tr>
<tr>
<td>26. The guidebook provided is very helpful and enriching.</td>
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<td></td>
</tr>
<tr>
<td>27. The idea of providing learning at the beginning of every lesson in the textbooks activities (e.g. the TRY THIS section) is a very enriching idea for the students.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Part III: Example learning activity

Kindly spend a few minutes to answer this section, please.

From the following list of objectives related to the learning of fractions, please choose one of them based on your previous teaching experience and design/describe an appropriate learning activity.

i. Class PP-A8 → Halves: meaning (in context)

ii. Class 1-A10 → Fractional parts: simple denominators

iii. Class 2-A8 → Simple fractions: modeling numerators/denominators

iv. Class 4-A4 → Equivalent fractions (with model)

v. Class 4-A5 → Compare & order fractions

vi. Class 5-A5 → Fraction meaning: Division

vii. Class 5-A6 → Rename Fractions: With and without models (conceptual)

viii. Class 5-A7 → Compare & Order fractions (using reasoning)

<table>
<thead>
<tr>
<th>Class:</th>
<th>Objective:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Learning activity description</td>
</tr>
</tbody>
</table>
APPENDIX C: SAMPLE LESSON PLAN SUGGESTED IN THE TEXTBOOK

3.1.2. Fractions as Division

Try This
Bipy cooked a pot of soup for \( \frac{1}{2} \) hr.
She stirred it every \( \frac{1}{4} \) hr.
A. How many times did she stir the soup?

B. Explain how the problem in part A relates to the division meaning of a fraction.

Examples

**Example 1: Rounding Fractions as Whole Numbers**

Rounding each fraction as a whole number:

**Solution**

- **Thinking**
  - To make 1 whole, I divided the 15 halves by 5 to see how many whole halves there were.
  - I divided the 20 halves by 4 to see how many whole halves there were.

- **Solution**
  - \( \frac{15}{2} = 7 \) and \( \frac{20}{4} = 5 \)

**Example 2: Rounding Fractions as Mixed Numbers**

Rounding each improper fraction as a mixed number:

**Solution**

- **Thinking**
  - I knew that 12 \( \div \) 2 = 6 as I rounded 12 as
  - I knew the remainder of 2 meant there were 2 eighths left over.

Practising

1. What fraction of each picture?

a) [Picture of fractions]

b) [Picture of fractions]

c) [Picture of fractions]

d) [Picture of fractions]

e) [Picture of fractions]

6. Pads of paper are sold in packages of 4. Pema used 16 pads. How many full and part packages did he use?

7. Dorji showed that \( 3 \div 4 = \frac{3}{4} \) by doing this:
   - He thought of \( 3 \div 4 = 3 \) whole items shared among 4 people.
   - He gave each person \( \frac{1}{2} \) of each item, so each got \( \frac{3}{4} \) of one item.
   a) Do you agree with his method?
b) Use Dorji’s method to show that \( 6 \div 5 = \frac{6}{5} \)

8. a) Four different fractions were all renamed as \( \frac{2}{4} \). What might they have been?
b) Could more than one fraction be renamed as \( \frac{1}{2} \)? Explain your thinking.

9. Dupho wrote \( \frac{3}{25} = \frac{6}{9} \).
   a) Is he correct? How do you know?
b) [Picture of fractions]

10. Why do you think fractions are about division?
### Appendix D1: Understanding framework

<table>
<thead>
<tr>
<th>Indicators</th>
<th>What the teacher can do</th>
<th>Remarks</th>
<th>What students can do</th>
</tr>
</thead>
</table>
| Use learning activities that explore and scaffold construction of conceptual schema: | - Horizontally between students’ reality (i.e., daily-life activities or situations that can be readily imagined by the students) and mathematical activities  
- Vertically between mathematical ideas of varying levels of abstraction or sophistication | Participate in activities in which they identify and describe the developing horizontal and vertical connections within their own conceptual schema |                                                                                                                          |

| Non indicators  | Use of learning activities, which do not incorporate opportunities to explicitly explore the connections between mathematical ideas | Passively listen to teacher explanations  
Does not describe the vertical and horizontal connections they perceive between mathematical ideas |                                                                                                                          |

Green (dark shading) → Teacher’s action aligning with the curriculum intention  
Orange (dark) → Students’ engagement aligning with the curriculum intention  
Green (light shading) → Teacher’s action not aligning with the curriculum intention  
Orange (light) → Students’ engagement not aligning with the curriculum intention
### Appendix D2: Reasoning framework

<table>
<thead>
<tr>
<th>Indicators</th>
<th>What the teacher does?</th>
<th>Remarks</th>
<th>What students do?</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Descriptions</strong></td>
<td><strong>Remarks</strong></td>
<td><strong>Descriptions</strong></td>
<td></td>
</tr>
<tr>
<td>Indicators</td>
<td>Use thought provoking learning activities which encourage students to explain their thinking and mathematical ideas.</td>
<td>Explain their thinking and mathematical activity.</td>
<td>Flexibly use of written and spoken language as well as symbolic, iconic and enactive representations.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Integrate a range of written and spoken language and representations (enactive, iconic and symbolic) to express mathematical ideas.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Counter-indicators</td>
<td>Emphasis correct answers over the processes by which the answers were created.</td>
<td>Focus on achieving correct answers without explaining or justifying the processes used.</td>
<td>Use rules and formulae without explaining why and how these rules and formulae work.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Encourage reliance on ready-made rules and formulae without helping students to understand the origin of these rules or formulae.</td>
<td></td>
<td>Do not flexibly use a range of language and symbolic, iconic or enactive representations.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Primarily uses symbolic representations to express mathematical ideas.</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Purple (dark shading) → Teacher’s action aligning with the curriculum intention

Purple (light shading) → Teacher’s action not aligning with the curriculum intention

Maroon (dark) → Students’ engagement aligning with the curriculum intention

Maroon (light) → Students’ engagement not aligning with the curriculum intention
Appendix D3: Context framework

<table>
<thead>
<tr>
<th>Indicators</th>
<th>What the teacher does?</th>
<th>What the students do?</th>
</tr>
</thead>
<tbody>
<tr>
<td>____</td>
<td>Contextualise learning activities based upon students’ reality (daily-life activities or situations that can be readily imagined).</td>
<td>Explain their thinking and mathematical activity. Flexibly use of written and spoken language as well as symbolic, iconic and enactive representations.</td>
</tr>
<tr>
<td>Non-indicators</td>
<td>Do not base learning activities on students’ reality, and instead relies upon use of de-contextualised mathematical learning activities.</td>
<td>Focus on achieving correct answers without explaining or justifying the processes used. Use rules and formulae without explaining why and how these rules and formulae work. Do not flexibly use a range of language and symbolic, iconic or enactive representations.</td>
</tr>
</tbody>
</table>

Blue (dark shading) ➔ Teacher’s action aligning with the curriculum intention

Red (dark) ➔ Students’ engagement aligning with the curriculum intention

Blue (light shading) ➔ Teacher’s action not aligning with the curriculum intention

Red (light) ➔ Students’ engagement not aligning with the curriculum intention
# APPENDIX E: PARTICIPANTS’ LESSON PLANS

**Appendix E1: Phurba’s lesson plans**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Lesson Objectives</th>
<th>Information Input</th>
<th>Learning Activities</th>
<th>Lesson Closure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Meaning of fractions</strong></td>
<td><strong>Lesson One</strong> -represent the fraction using a shape, group and length correctly; -write a fraction for the given picture</td>
<td>-explanation using diagrams/students/number line on the concept of fractions - use number line to introduce the meaning.</td>
<td>i. Draw a picture to represent the fraction ii. Represent in fraction symbolically for the given pictures</td>
<td>Not indicated</td>
</tr>
<tr>
<td><strong>Fractions as division</strong></td>
<td><strong>Lesson Two</strong> -write fractions as whole number correctly -rename fractions as mixed number correctly -able to develop the relationship between fractions and divisions</td>
<td>-Discuss TRY THIS activity with the class</td>
<td>Pair work -write it as mixed number: 54/5; 18/4 -write it as a whole number: 21/7; 25/5 -draw a diagram to show: 3 ÷ 4 is ¾</td>
<td>No proper closure</td>
</tr>
<tr>
<td><strong>Equivalent fractions</strong></td>
<td><strong>Lesson Three</strong> -be able to create equivalent fractions for the given fractions correctly</td>
<td>-Revision on previous lesson -Explantion on fractions as division using diagrams - a brief revision on previous lesson - Try This activity - Guided demonstration on how to create equivalent fractions through drawing diagrams - explanation on how to find equivalent fractions using diagrams followed by rules (multiplication and division)</td>
<td>Draw a diagram to show: 3 ÷ 4 is ¾ 3 ÷ 2 is 2 ½</td>
<td>No closure</td>
</tr>
<tr>
<td><strong>Lesson Four</strong> -able to create equivalent fractions correctly</td>
<td></td>
<td>-recapitulate the previous lesson on finding equivalent fractions -Explanation on finding equivalent fractions using division method</td>
<td>Creating equivalent fractions a. 4/8 b. 3/9 c. 20/30</td>
<td>How do we create equivalent fractions?</td>
</tr>
<tr>
<td>Topic</td>
<td>Lesson Objectives</td>
<td>Information Input</td>
<td>Learning Activities</td>
<td>Lesson Closure</td>
</tr>
<tr>
<td>-----------------------</td>
<td>------------------------------------</td>
<td>------------------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>Comparing of fractions</td>
<td>Lesson Six -able to compare and order fractions correctly</td>
<td>-TRY THIS activity -comparing of fractions using diagrams - Introduction of rules on comparing fractions with: i. same denominators and same numerators ii. different denominators and numerators</td>
<td>Compare a pair of fractions, e.g., 4/9 and 5/9 3/5 and 3/7; 2/3 and ¼</td>
<td>No closure</td>
</tr>
<tr>
<td>Ordering of fractions</td>
<td>Lesson Seven -Compare fractions with different denominator and numerators -Compare improper fractions</td>
<td>TRY THIS activity (In a test Sonam scored 3/5 of the total marks and Pema scored 4/8 of the total marks. Who scored more marks? -explanation on comparing of fractions using number line</td>
<td>Compare a pair of fractions, using number lines e.g., 1/4 and 7/9 3/5 and 3/7; 2/3 and ¼</td>
<td>Q.2</td>
</tr>
<tr>
<td></td>
<td>Lesson Eight -able to compare and order fractions correctly</td>
<td>TRY THIS activity: Sonam has 11/5 apples and Karma has 7/4 apples. Who has more apples?</td>
<td>Compare and order fraction least to greatest; i. 4/5, 3/7, 2/4; ii. 4/8, 11/5, 5/6 Group activity: Which fraction is greater i. ½ or 1/6 ii. 2/20 or 25/26</td>
<td>Q.3 &amp; 4 (page 90)</td>
</tr>
</tbody>
</table>
## Appendix E2: Norbu’s lesson plans

<table>
<thead>
<tr>
<th>Topic</th>
<th>Lesson Objectives</th>
<th>Information Input</th>
<th>Learning Activities</th>
<th>Lesson Closure/</th>
</tr>
</thead>
</table>
| Meaning of fractions | Lesson One  
- draw a picture to represent a fraction given for a shape, group or a length correctly  
- tell and write a fraction for the given picture | - questions to asked based on the content of the chart paper pasted on the board  
- explain the problem given on the chart paper with the help of drawing diagrams and shading  
- Demonstration on folding piece of paper to introduce a concept of fraction  
- display a concept of fractions through diagrams as a part of whole object and a part of whole set | (practicing and applying) | Revise the meaning of fractions |
| Fractions as division | Lesson Two  
- develop the relationship between fractions and division  
- change an improper fraction to a mixed number | - Try This activity in pair  
- Explain the problem on the board as 12 ÷ 3  
- explain the problem 2 ÷ 4 = ½ using two identical circles and four students  
- explain the conversion of improper to mixed fractions with the help of diagrams and number line | Q.1a and Q.4 from the textbook | solve and explain a.  
¾ = 3 ÷ 4  
b. Q.1b, Q 2 & Q. 9 |
| Lesson Three  
- change an improper fraction to a mixed numbers  
- link concrete materials and/or pictorial representations to symbols to develop understanding | - recapitulate previous lesson on how and why 5/2 as 2 ½ with the help of diagrams  
- explanation on conversion of improper to mixed numbers with the help of diagrams | Q.3 from the textbook | |
<table>
<thead>
<tr>
<th>Equivalent fractions</th>
<th>Lesson Four</th>
<th>Lesson Five</th>
<th>Lesson Six</th>
<th>Lesson Seven</th>
<th>Lesson Eight</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-Develop an understanding of renaming fractions using concrete materials and/or pictorial representations first and then link to the symbolic -understand equivalent fractions as the same region or group partitioned in different ways.</td>
<td>-try This with a partner -explain finding equivalent fraction using multiplication rules</td>
<td>-understand equivalent fractions as the same region and the group partitioned in different ways -understand equivalent fractions in order to make sense of fractions involving larger numbers</td>
<td>-Try This activity -Introduce the concept of comparing a pair of fractions with the help of diagram and shading (i.e. 4/5 and 2/5)</td>
<td>-compare fractions with the same numerator -compare fractions with the same denominator</td>
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<tr>
<td></td>
<td>Q.2. to solve 20/4 as 20 ÷ 4 with diagrams (textbook)</td>
<td>Q.1b from the textbook</td>
<td>Q.1a from the textbook</td>
<td>Q.3 from practice and applying</td>
<td>Q.5a on page 90</td>
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<tr>
<td></td>
<td>Q2. solve and show 10/2 = 5 -17/3 = 5 2/3</td>
<td>Q.4b</td>
<td>Q.2 as homework</td>
<td>Q.4 as a homework</td>
<td>Q. 6 (practice and applying)</td>
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### Comparing of fractions

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<tr>
<th>Lesson Six</th>
<th>Lesson Seven</th>
<th>Lesson Eight</th>
</tr>
</thead>
<tbody>
<tr>
<td>-develop and use benchmarks to compare fractions -compare fractions with the same denominators and the same numerator</td>
<td>-demonstration on comparing of fractions using fraction strips and number line -conduct Try This from the textbook -explain on comparing of improper fractions</td>
<td>-compare fractions with the same numerator -compare fractions with the same denominator</td>
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<tr>
<td>Q.4b</td>
<td>Q.3 from practice and applying</td>
<td>Q. 6 (practice and applying)</td>
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Appendix E3: Sangay’s lesson plans

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<thead>
<tr>
<th>Topic/ Lesson</th>
<th>Lesson Objectives</th>
<th>Information Input</th>
<th>Learning Activities</th>
<th>Lesson Closure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meaning of fractions</td>
<td>Lesson One</td>
<td>- tell the meaning of fractions correctly after teacher’s explanation</td>
<td>-Revisit on concept of fractions by asking a few general questions</td>
<td>Two group Activities: i. Paper folding as per the - instruction ii. Shade and represent the given fractions on the worksheet provided</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- relate fraction as a part of a shape and a part of a group</td>
<td>- Teacher’s demonstration on the concept of fractions through folding a piece of paper in halves</td>
<td>- recapitulate the main points discussed through question-answer session</td>
</tr>
<tr>
<td></td>
<td>Lesson Two</td>
<td>- Conduct Try This activity</td>
<td>- pair work by providing three pieces of paper to divide among themselves equally to represent 3/2 as 1 ½</td>
<td></td>
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<tr>
<td>Fractions as division</td>
<td></td>
<td>- Explain the concept of fraction as division using 3/2 as an example</td>
<td>- represent 5 ÷ 6 in diagrams individually</td>
<td></td>
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<tr>
<td></td>
<td>Lesson Three</td>
<td>- demonstrate the concept with the help of diagrams</td>
<td>- Display picture model on the chart paper</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- represent equivalent fraction using pictures</td>
<td>-Try This activity</td>
<td>-Q. 2 &amp; 5 (textbook) homework</td>
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</tr>
<tr>
<td></td>
<td>- create equivalent fractions symbolically</td>
<td>-Explain the concept of equivalent fractions with the help of diagrams</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lesson Four</td>
<td>- explain the rules of creating equivalent fractions through multiplication and division</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>- compare fractions with the same denominator and same numerator correctly after teacher’s explanation</td>
<td>- revisit previous lesson through close questioning session: i. what did we learn in the last class? ii. How can we create equivalent fractions?</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lesson Five</td>
<td>- explain the concept of comparing fractions in terms of numerators and denominators</td>
<td>Individual activity on comparing</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- compare fractions with different numerator and denominator correctly</td>
<td>- demonstration with the help of diagrams</td>
<td>-revisit the lesson by asking few questions</td>
<td></td>
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</table>

Appendices 300
<table>
<thead>
<tr>
<th>Topic/</th>
<th>Lesson Objectives</th>
<th>Information Input</th>
<th>Learning Activities</th>
<th>Lesson Closure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordering of fractions</td>
<td>Lesson Six- - order three sets of fractions from least to greatest -Order fractions from the least to greatest</td>
<td>-revisit previous lesson close questioning session: i. what did we learn in the last class? ii. How can we compare fractions? - explain the concept of ordering of fractions</td>
<td>Group activity on ordering of fractions from least to greatest i. 7/5/, 25/8, 26/5, 9/4, 19/3 ii. ½, 1/3, 5/8, 2/5, 1/4</td>
<td></td>
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<tr>
<td>Topic</td>
<td>Lesson Objectives</td>
<td>Information Input</td>
<td>Learning Activities</td>
<td>Lesson Closure</td>
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<tr>
<td>Meaning of fractions</td>
<td>Lesson One</td>
<td>- Revisit on the concept of fractions by asking a few general questions: i. what is fractions? Examples? ii. Can you draw a fraction on the board? - Teacher’s demonstration on the concept of fractions through folding a piece of paper in halves - invite a group of students to explain the concept of fractions in thirds</td>
<td>Two group Activities: i. Paper folding as per the - instruction ii. Shade and represent the given fractions on the worksheet provided</td>
<td>Recapitulate the main points discussed through question-answer session</td>
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<tr>
<td></td>
<td>Lesson Two</td>
<td>- Revision on previous lesson - explain the concept on fractions as division using: i. paper ii. students iii. fractions models</td>
<td>- call six students to demonstrate sharing five pieces of papers - provide two pieces of papers to divide among four students Q. 3a and 3b from the textbook</td>
<td>- Revise the topic once again and homework from the textbook (Q. 2, 4, &amp; 5, page 81)</td>
</tr>
<tr>
<td>Fractions as division</td>
<td>Lesson Five</td>
<td>- revision on equivalent of fractions - explain the concept of comparing of fractions screening examples on the LCD projector</td>
<td>- Conduct of Try This activity from the textbook (page 87)</td>
<td>Revision of the topic taught</td>
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<tr>
<td>Comparing of fractions</td>
<td>Lesson Six</td>
<td>- understand the meaning of ordering of fractions - tell the strategies of solving the questions</td>
<td>- Q. 2a 7 b from the textbook (page 90)</td>
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<tr>
<td>Ordering of fractions</td>
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<td>- understand the meaning of ordering of fractions</td>
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### Appendix E5: Tempa’s lesson Plans

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<tr>
<td>Equivalent</td>
<td>Lesson Three&lt;br&gt;- find equivalent fractions of a given fraction using pictorial representations&lt;br&gt;- explore equivalent fraction by multiplying numerator and denominator&lt;br&gt;- explore equivalent fraction by subdividing equally</td>
<td>- conduct of TRY This activity in pair with the help of following questions:&lt;br&gt;i. How do you know that the amount for each day is only a fraction of a page?&lt;br&gt;ii. How do you know the amount for each day is more than half?&lt;br&gt;iii. How did you get 6/9?&lt;br&gt;- Demonstration on the concept of equivalent fractions through folding paper&lt;br&gt;- Presentation on definition of equivalent fractions</td>
<td>- No specific activity for students indicated in the plan</td>
<td>- revisit the Try This activity</td>
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<td>fractions</td>
<td>Lesson Four&lt;br&gt;- create equivalent fractions of a given fraction by multiplying and dividing the numerator and denominator</td>
<td>- presentation of self-designed activity integrated with the concept of equivalent fraction&lt;br&gt;- Discussion on the concept of finding equivalent fractions using multiplication and division methods</td>
<td>- no specific activity mentioned</td>
<td>- Revisit the lesson once again by asking question</td>
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### APPENDIX F: FREQUENCY OF IMPLEMENTATION TO FOUR SUGGESTED LESSON COMPONENTS

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<tr>
<th>Teachers/components</th>
<th>Topics</th>
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</table>
I have completed my Bachelor of Education in Pulo College of Education in the year 2001. For the last 6 years, I have been teaching mathematics new curriculum and still I am continuing my teaching like same. Although I have been teaching this new curriculum, I didn’t get a proper orientation to implement this new text to the learners. As far as possible, I tried and seeked help from the people who were oriented, but that was not enough for me to apply practically to the students. I didn’t know the actual motive and the objective of the coming up of this new curriculum mathematics. I just thought that it is more of English (theory) than the practical.

This year, I was lucky to be met with Mrs. Phumzile. Mrs. Phumzile was my mathematics teacher when I was in the diploma my mathematics teacher. She was a great honour for me to have Mrs. Phumzile as my student affairs in Pulo College of Education and currently doing her PhD, a researcher of School of Mathematics Science & Technology Education in Queensland University of Technology, Australia for the past three months.
She came to my school in the beginning of this year to do a research on the gap between the curriculum intent and practice in Bhutanese Primary School.

As soon I accepted to be one of her participants, it is because I knew her from my college times and it was a good opportunity for me to learn from her as she is the right person for this new curriculum. During her stay here I learned a lot, she has been observing my lesson in every teaching and I got all the feedback. I was able to clarify my doubts with her and I knew how to teach this new curriculum. She even guided me to teach this new curriculum. She has been a role model for me. The new curriculum is based on the need of the students for the future curriculum and I still have a long way to go as there are many new things a mathematics teacher should learn.

She has really motivated me to learn and go ahead with this new curriculum. Therefore I would like to thank Mrs. Phuntscho Delma for her help and guidance in this new curriculum and next time working together in her team.
This is my 12th year in teaching Mathematics and for the last five years I have taught Mathematics in classes ranging from Year 5. The Mathematics curriculum in our country is a newly introduced mathematics curriculum. Although I have been teaching mathematics for a good number of years, yes I did not have a clear understanding on how the new curriculum should be taught. Besides the intention and its implementation of the new Mathematics curriculum I did not get any opportunity to reflect on the effectiveness of my strategies in helping the students to understand the mathematical concept more deeply and meaningfully. Therefore, I taught Mathematics without following the set principles and process of the new curriculum.

However, it was a great honor for me to have Dr. Phan Le Dinh, Dean of Student Affairs in the College of Education and currently a Researcher (PhD Candidate) at the Queensland University of Technology, Brisbane, Australia in my class for the last three months. During her stay, I got an opportunity to interact with her. She observed my lessons and we had a good time sharing ideas on professional development (PD). She shared her ideas on how the new mathematics curriculum was designed and the intended learning outcomes and how the curriculum should be taught to the students. She also clarified my doubts on many pressing issues relating to teaching of mathematics both on Pedagogy and the content. After learning more about the new mathematics curriculum, I came to realize that there are so many new things that a mathematics teacher should learn and that a Ministry of Education, whether at long or short term. Thus, she has really enriched the day of my knowledge on newly introduced mathematics curriculum.

In the conclusion, I can say that the Phan Le Dinh, Dean and finding from research has definitely filled the gap between curriculum intent and practice in Mathematics Primary school Mathematics classrooms and will benefit the Ministry of Education, Central education, General Science, Science and especially the Mathematics teachers and the students.